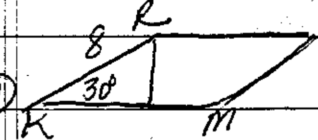
Geometry Team Solutions



44 ① Using 30-60-90 Δ ratios the height is 4 · 11 = 44

2.

8, 11

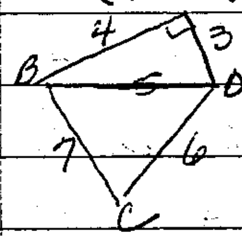
$$\frac{x-3}{3x-19} = \frac{4}{x-4}$$

$(x-3)(x-4) = 4(3x-19)$

$x^2-7x+12 = 12x-76$

$x^2-19x+88=0$

$(x-8)(x-11)=0 \quad x=8 \quad x=11$



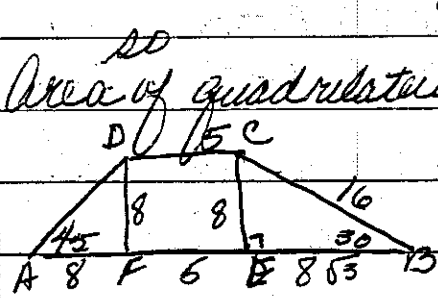
3 Since ∠BAD is right, you can establish that diagonal BD is 5 by the Pythagorean Theorem. Area of top Δ is 6

6+6√6 Using Heron's formula $s = \frac{1}{2}(5+6+7) = 9$

$\sqrt{9(9-5)(9-6)(9-7)}$

so

Area of quadrilateral is $6+6\sqrt{6}$ $\quad \sqrt{9 \cdot 4 \cdot 3 \cdot 2} = 6\sqrt{6}$

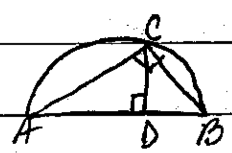


4. Using special Δ ratios the altitude to side $\overline{AB}$ is 8. $\overline{EB}$ is $8\sqrt{3}$. FE=5,

72+32√3 AF=8 $A = \frac{1}{2}(8)(5 + (8+5+8\sqrt{3}))$

$4 \ (18+8\sqrt{3})$

$72+32\sqrt{3}$



5. Since CD is the hypotenuse of the inscribed

10 right triangle CD is the geometric mean between AD and DB. AD = 5 DB = 25-5 = 20

$$\frac{x}{5} = \frac{20}{x} = x^2 = 100$$

$$x = 10$$

(15, -8 6. G, the midpoint of AB is $\left(\frac{-3+5}{2}, \frac{2+12}{2}\right) = (1,7)$ Using $y = mx+b$

line $\overleftrightarrow{HG}$ has equation $H(10,2)$ $\quad 7 = \left(\frac{7-2}{1-10}\right)(1) + b$

$$7 = -\frac{5}{9} + b$$

The line parallel has slope $-\frac{5}{9}$ $\quad 7\frac{5}{9} = b$

$2 = -\frac{5}{9}(-3) + b \quad 2 = \frac{5}{3} + b \Rightarrow b = \frac{1}{3}$

over

#6

Take the intersection of $\overleftrightarrow{BH}$ and $\overleftrightarrow{AC}$

$y = -5/9\,x + 1/3$ $\qquad$ $\overleftrightarrow{BH} = \frac{10}{-5}$ slope

$12 = -2(5) + b$

$12 = -10 + b$

$22 = b$ $\qquad y = -2x + 22$

Solving system $\quad 9y = 5x + 3$

$y = -2x + 22$

$x(-9)$ $\quad -9y = 18x - 198$

$9y = -5x + 3$

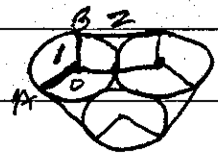
$195 = 13x$

$15 = x$ $\qquad$ putting this in the second equation

$y = -30 + 22$

$y = -8$ $\qquad$ Solution $(15, -8)$

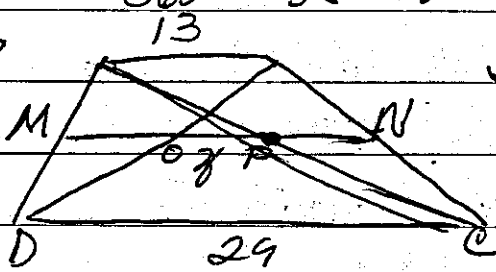
#7 $\quad 2\pi + 6$



Radii are $\perp$ to tangents $R = 1$ each tangent segment $= 2$. Each circle has a rt $\angle$ and 2 120° angles. There are three arcs the length of AB

$\frac{120}{360} = \frac{1}{3}(2\pi)$ or $\frac{2}{3}\pi$ $\qquad 3\left(\frac{2}{3}\pi + 2\right) = 2\pi + 6$

#8 $\quad$ 8



In the trapezoid MN is a median so $\overline{MO}$ is a segment that joins the midpoints of the sides of a triangle and measures $6.5 = x$ $\quad y$ also equals 6.5. Since $MN = 21$ and $21 - 2(6.5) = 8$

$6\sqrt{3}$ #9 If the ratios of the areas is $\frac{4}{3}$ then the ratio of the sides is $\frac{\sqrt{4}}{\sqrt{3}}$

$\frac{2}{\sqrt{3}}$ $\quad$ Since $6x + 5x + 5x + 4x + 4x + 3x = 54$

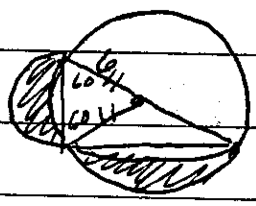
$\frac{2}{\sqrt{3}} = \frac{12}{x}$ $\qquad 27x = 54$

$x = 2$ and the longest side $= 12$

$2x = 12\sqrt{3}$

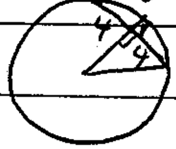
$x = 6\sqrt{3}$

#10 Since $AC = 6$ the area of the semicircle there
$18\sqrt{3}$ is $\frac{9\pi}{2}$. Since the ratio of AB to AC is 2:1 we have a
30-60-90 Δ and $CB = 6\sqrt{3}$. Therefore the semicircle
on $\overline{CB}$ has radius $3\sqrt{3}$ and area $\frac{27\pi}{2}$



To find the area of the smaller segment find the area of sector $\frac{60}{360} = \frac{1}{6} \cdot \frac{36\pi}{1} = 6\pi$ and subtract the area of $\Delta FAC =$
$\frac{1}{2}(6)3\sqrt{3} = 9\sqrt{3}$ Now we have $6\pi - 9\sqrt{3}$
subtracted from $\frac{9\pi}{2} - (6\pi - 9\sqrt{3}) = 9\sqrt{3} - 1.5\pi$ area of small moon
The area of the larger segment is $\frac{1}{3} \frac{36\pi}{1} = 12\pi - \Delta$
The second Δ has area $\cong$ to first segment area $= 12\pi - 9\sqrt{3}$.
Moon area $= \frac{27\pi}{2} - (12\pi - 9\sqrt{3}) = 9\sqrt{3} + 1.5\pi$. Thus the
sum of the shaded regions is $9\sqrt{3} - 1.5\pi + 9\sqrt{3} - 1.5\pi = 18\sqrt{3}$

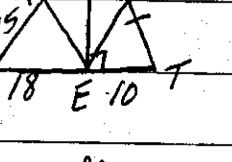
11
17π



Because the radius is $\perp$ to the chord it bisects the chord thus $(r-1)^2 + 4^2 = r^2$

$$r^2 - 2r + 1 + 16 = r^2$$
$$17 - 2r = 0$$
$$17 = 2r = d \quad C = 17\pi$$

67.5° 12

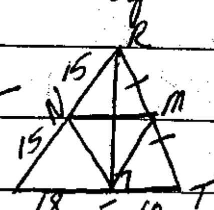


$$4y + 3y + y = 360$$
$$8y = 360$$
$$y = 45$$

$$x = \frac{4y - y}{2} = \frac{3y}{2} = \frac{3(45)}{2}$$
$$x = \frac{135}{2} = 72.5^\circ$$

$MN = 14$
$MT = 13$
$EN = 15$

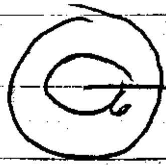
(13)



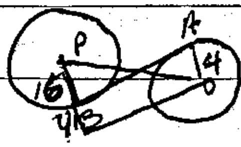
$MN = 14$ because it joins the midpoints of the sides of a Δ

$\overline{RE}$ measures $30^2 - 18^2 = \sqrt{900 - 324} = \sqrt{576} = 24$

Because $24^2 + 10^2$
$576 + 100 = \sqrt{676} = 26 = MT$ Since EM is a median from
[illegible] of a rt. Δ it measures $\frac{1}{2} MT = 13$

(14) 

Area $= 36\pi$ $\frac{1}{2}\,36\pi = 18\pi$ $\sqrt{18} = 3\sqrt{2}$ = radius of inner circle. So $6-3\sqrt{2}$ is width of border

15 

$\overline{PB}$ and $\overline{OA}$ are $\perp$ to $\overline{AB}$. Extending PB to form a right $\triangle$

$10^2 + AB^2 = 15^2$

$100 + AB^2 = 225$

$AB^2 = 125$

$AB = 5\sqrt{5}$