Mu Alpha Theta National Convention: Seattle, 1997

Alpha School Bowl Test

1. A sector with an acute central angle of 30º is cut from a circle of radius r. A circle of radius R is then circumscribed about that sector. What is the ratio of R to r?

2. A division problem is discovered in an old book on base arithmetic, but many of the digits are illegible. What was the quotient in the problem?

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3. Consider an acidic solution. If it were somehow possible to extract a liter of pure acid, the resulting solution would be only 12% acid. If a liter of water were added to the original solution, it would reduce its acidity to 25%. What was the volume of the original solution, in liters?

4. Determine the smallest number consisting of 3's and 4's but no other digits, such that both the number and the sum of its digits is divisible by both 3 and 4.

5. If A is the denominator of the answer to problem 1, B is the answer to problem 2, C is the numerator of the answer to problem 3, and D is the answer to problem 4, evaluate

$$D - ABC$$

6. Jack and Jill went for a 6 hour walk. The walked along a level road until they came to a hill. They climbed the hill, then immediately turned around and retraced their path to their house. If their walking speed is 4 kmph on level ground, 3 kmph uphill, and 6 kmph downhill, what was the total distance, in kilometers, they traveled that day?

7. Two complex numbers, A and B, sum to $5-3i$. Their product is one eighth the result of dividing A by the square of B. Determine the sum of all possible values of A.

8. Simplify: $\left[(\log_a b)(\log_a c)\right]^{(\log_a \sqrt{b})(\log_c b)(\log_b a)}$

9. Bill and Tom play 5 games of pool. If Tom's probability of winning four games is the same as his probability of winning three games, what is the probability that Bill wins three games?

10. If A is the answer to problem 6, B is the real part of the answer to problem 7, C is the answer to problem 8, and D is the denominator of the answer to problem 9, substitute and simplify

$$(AB+D)C$$

11. What is the sum of the cubes of the roots of the equation $x^3 + 3x^2 + 4x + 5 = 0$?

12. A each member of a boy scout troop pitches in to buy a canoe. If the troop had had 4 more members, the cost per person would have been 3 dollars less. If the troop had had 2 fewer members, the cost per person would have been 2 dollars more. What was the price of the canoe?

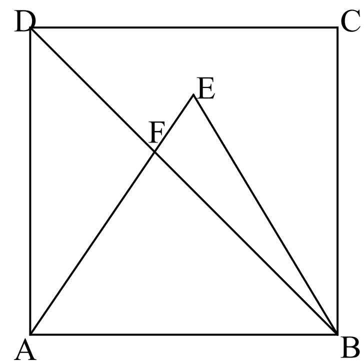
13. Simplify: $\sqrt{31+12\sqrt{3}}$

14. If I am to paint each of the sides of a regular tetrahedron one of four colors, how many distinguishable patterns can I produce?

15. If A is the answer to problem 11, B is the answer to problem 12, C is the greatest integer less than the answer to problem 13, and D is the answer to problem 14, evaluate

$$ADC + B$$

16. In the diagram, diagonal DB of square ABCD intersects side AE of equilateral triangle ABE at F. What is the ratio of the area of triangle BEF to that of square ABCD?



17. To the nearest thousandth, $\log_{10} 2 = .301$ and $\log_{10} 3 = .477$. What should the value of $\log_5 10$ be, to the nearest thousandth?

18. Consider a hollow glass cube 6 centimeters on a side which contains two spiders. One of the spiders is in the center of a wall, 1 cm from the bottom of the cube. The other is on the opposite wall, 1 cm from the top of the cube and 1 cm from an adjoining wall. What is the minimum distance a spider would have to travel to get to the other's position?

19. Three people are shown 3 red hats and 2 black hats that are to be placed upon them. They are seated one behind the other, so that person A can see persons B and C, person B can see person C, and person C can see no one, and then the hats are placed upon them, one per person. Person A is then asked whether he knows what color his hat is. If he does not, person B is then asked what color her hat is, and if she does not know, then person C will be asked whether she knows her hat's color. What is the probability that person C is asked her hat's color?

20. If A is the denominator of the answer to problem 16, B is the answer to problem 17, C is the greatest integer less than the answer to problem 18, and D is the answer to problem 19, evaluate

$$\frac{AC}{D} - B$$

21. For how many real numbers n is $(n-2i)^5$ also a real number?

22. If $x+\frac{1}{x}=2\cos\theta$, what is the value of $x^5+\frac{1}{x^5}$ when θ is equal to 30º ?

23. A field in the shape of a right triangle with legs of 4 and 2 has a post at the midpoint of each side. A sheep is tethered to each of the leg posts and a goat to the post on the hypotenuse. The ropes are just long enough to let each animal reach the two adjacent vertices. What is the total area that a sheep can reach but the goat cannot?

24. Find a three digit number which is equal to the sum of the cubes of its digits.

25. If A is the answer to problem 21, B is the answer to problem 22, C is the answer to problem 23, and D is the answer to problem 24, evaluate

$$(D-AC)B$$

26. How many positive integers are less than and relatively prime to 588?

27. A natural number, n, can be expressed as a three digit number in base x. When these three digits are reversed, and regarded as a base x number, it has twice the value of n. Find the value of n, expressed in base 10, such that the sum of the digits in base x is a minimum.

28. For how many values of n from one to ten inclusive is it impossible to distribute ten pennies in n Styrofoam cups so that each cup contains an odd number of pennies?

29. What is the volume of the pyramid defined by the points (4, 5, 6), (-3, -1, 5), (7, 4, -1), and (3, 3, 3)?

30. If A is the answer to problem 26, B is the answer to problem 27, C is the answer to problem 28, and D is the answer to problem 29, evaluate

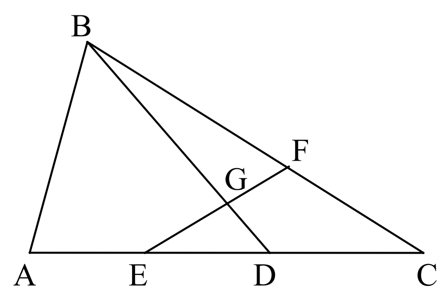
$$A - BCD$$

31. In circle O, there are three equally spaced parallel chords, A, B, and C on the same side of the center, whose lengths are 20, 16, and 8. What is the radius of the circle?

32. Two men sit down to play cards. The first wager is one cent, the second two cents, doubling with each bet. The gambling ends when one of the men, who brought only $6.01 is cleaned out. If this happens in the fewest possible number of bets, what is the largest number of cents in the loser's possession that night?

33. On each face of a cube, you are to draw a line connecting opposite vertices. How many distinguishable patterns could be thus produced?

34. In triangle ABC, $AE = ED = DC \text{ and } BF = FC$. What is the ratio of the area of triangle EGD to that of triangle ABC?



35. If A is the answer to problem 31, B is the answer to problem 32, C is the answer to problem 33, and D is the answer to problem 34, evaluate

$$\frac{AC}{D} + B$$

36. In how many ways can one trace a path in 3-space from the origin to the point (5, 5, 5) where each leg of the path is between adjacent lattice points, and the direction of travel is always parallel to either the x, y, or z axes?

37. Evaluate $\sqrt[3]{1+\frac{\sqrt{5}}{2}}+\sqrt[3]{1-\frac{\sqrt{5}}{2}}$

38. Determine the sum of all values of θ $(0 \le \theta < 2\pi)$ for which $3\tan^4 4x - 4\sec^2 4x + 5 = 0$.

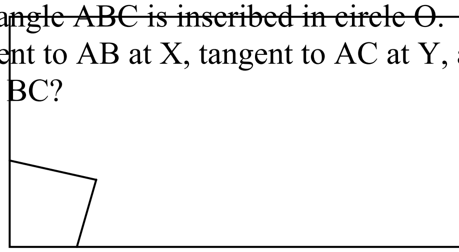
39. How many two by two matrices are their own inverses?

40. If A is the answer to problem 36, B is the answer to problem 37, C is the coefficient of π in the answer to problem 38, and D is the answer to problem 39, evaluate

$$A - \frac{C}{B^3} - \frac{1}{D}$$

41. Determine the maximum area in the corner of a rectangular room that can be enclosed by two screens 6 feet wide.

42. Equilateral triangle ABC is inscribed in circle O. Circle P is entirely contained within circle O, and is tangent to AB at X, tangent to AC at Y, and tangent to circle O at Z. What is the ratio of XY to BC?



43. A student miscopies an equation of the form $x^2 - Ax + B = 0$, transposing the digits of B as well as the plus and minus signs. One of the roots he got was the same as one of the roots of the original equation. What was the root?

44. In triangle ABC, AD and AE trisect angle BAC. If BD=2, DE=3, EC=6, and BC=11, what is the length of the shortest side of the triangle?

45. If A is the greatest integer less than the answer to problem 41, B is the answer to problem 42, C is the answer to problem 43, and D is the greatest integer less than the answer to problem 44, evaluate

$$ABCD$$

46. In the game “subtract a square”, a positive integer is written down and two players alternately subtract squares from it until a player is able to leave zero, in which case he is the winner. What is the optimal number for the first player to subtract, if the original number is 29?

47. Three sides of a regular dodecahedron are to be painted red, while the others are to be painted green. How many distinguishable patterns could this painting produce?

48. Four castaways collect coconuts on a desert island to divide amongst themselves the next day. Being distrustful, each of them gets up during the night, to hide his share of the coconuts, unaware that the others are doing the same. Each of them in turn divides the pile into 4 equal shares, leaving one coconut, which they give to a monkey. They then hide one of the four equal shares and return to bed, allowing the other castaways to do the same. In the morning, they divide the pile between themselves, discovering that there is an extra coconut, which they give to the monkey. What is the smallest number of coconuts they could have collected?

49. Triangle ABC is equilateral with side length 6. If circles of radius 4 are drawn centered at the vertices A, B, and C, what is the greatest distance between a point of intersection of two of the circles and a point of intersection of a different pair of circles?

50. If A is the answer to problem 46, B is the answer to problem 47, C is the answer to problem 48, and D is the greatest integer less than the answer to problem 49, evaluate

$$C - ABD$$