

1992 NATIONAL MU ALPHA THETA CONVENTION ALPHA BOWL

ALPHA BOWL #1

If $0 \leq \theta \leq 4\pi$, find, in radians, all values of θ which satisfy:

$$\log_{\frac{1}{2} \sin 2\theta} \sin \theta = \frac{1}{2}$$

ALPHA BOWL #2

For what real number(s) m is the graph of $x^2 + y^2 - 4x - 6y = m - 13$ tangent to the graph of $x + y = 5 + \sqrt{2m}$?

ALPHA BOWL #3

Solve for x on \mathbb{R} :
$$\sqrt[4]{x^3} + 2\sqrt{x} - x = 0.$$

ALPHA BOWL #4

Three of the elements of the solution set of the simultaneous system:

$$x^{x+y} = y^4, \quad y^{x+y} = x$$

are ordered pairs of integers, (x,y) . Find these three ordered pairs.

ALPHA BOWL #5

Find the sum of the infinite series:

$$\sum_{n=1}^{\infty} \frac{n^2}{3^{n-1}}$$

ALPHA BOWL #6

Two fair dice are tossed five times. Find the probability that the sum of 7 will show on the first three tosses and will not show on the other two.

ALPHA BOWL #7

The inverse of the function : $y = \log_2 \frac{2x-1}{2}$ is what?

ALPHA BOWL #8

If $8x - 11 \leq f(x) \leq 2x^2 - 3$ for all x on the interval $0 \leq x \leq 4$, then $\lim_{x \rightarrow 2} f(x) = ?$

ALPHA BOWL #9

Let the growth of a plant culture X (measured in grams per second) be directly proportional to the weight of the plant culture (measured in grams). If the weight of the culture at the beginning of the experiment ($t=0$) is 10 grams, and the weight after one second is 100 grams, find the difference between the instantaneous rate of growth when t is one second and the average rate of growth for the first second.

ALPHA BOWL #10

If $f(x) = e^{\frac{x^3}{3-x}}$, find where $f(x)$ is increasing.

ALPHA BOWL #11

Find the equation(s) of all horizontal asymptotes (if any) of: $f(x) = \frac{|x|}{|x| + x}$

ALPHA BOWL #12

A projectile is fired directly upward from the ground with an initial velocity of 112 ft/sec, and its distance above the ground after t seconds is $s(t) = 112t - 16t^2$ feet. What is the velocity at the moment of impact?

ALPHA BOWL #13

Find the value (as a real number) of $S = \sin^2(10^\circ) + \sin^2(20^\circ) + \sin^2(30^\circ) + \dots + \sin^2(90^\circ)$

ALPHA BOWL #14

After emptying their bank accounts, 5 children found that they had just enough to purchase a used moped. The oldest contributed one-half the amount brought by the others together, the second child contributed one-third the amount brought by the others together, the third contributed one-fourth the amount brought by the others together, the fourth contributed one-fifth the amount brought by the others together, and the youngest contributed \$14.70. Find the cost of the moped.

ALPHA BOWL #15

In a 3-dimensional rectangular coordinate system, find the total surface area of the solid defined by: $|x| + |y| + |z| \leq 1$