

3

**Mu Alpha Theta**  
**Probability, Permutations, and Combinations**

1) What is the probability that the product of 3 randomly selected integers will be odd?

- a)  $1/8$     b)  $1/4$     c)  $3/8$     d)  $1/2$     e)  $5/8$

2) A contractor wants to build houses in a new neighborhood. She has 5 floor plans to choose from and 6 different colors to paint the houses. How many different houses can she build (assuming that each house is painted with only one color)?

- a) 11    b) 25    c) 30    d) 36    e) 41

3) The average  $\bar{x}$  of the first 15 prime numbers is:

- a)  $\bar{x} \leq 14$     b)  $14 < \bar{x} \leq 16$     c)  $16 < \bar{x} \leq 19$     d)  $19 < \bar{x} \leq 22$     e)  $22 < \bar{x}$

4) What is the average of the first  $n$  odd numbers (starting at 1)?

- a)  $(n-1)/2$     b)  $n/2$     c)  $n$     d)  $n+1$     e)  $2n-1$

5) Evaluate: 
$$\frac{\binom{6}{4} \binom{7}{3} 3P_2}{\binom{5}{2}}$$

- a) 90    b) 168    c) 315    d) 336    e) 630

6) If  $x$  and  $y$  are randomly chosen integers between 1 and 10, inclusive, what is the probability that  $xy$  is prime?

- a)  $1/100$
- b)  $1/50$
- c)  $1/10$
- d)  $2/25$
- e)  $9/100$

7. A basketball coach wants to select 5 starting players from his team of 12. How many ways can he do this?

- a) 21
- b) 676
- c) 712
- d) 792
- e) 95040

8. If  $x + y$  is equal to the sum of the mean and the median of the following set of data and  $y$  is larger than  $x$ , find  $x$ .

DATA: 12, 5, 8, 9, 8, 11, 11, 4,  $x$ ,  $y$

- a) 3
- b) 5
- c) 6
- d) 7
- e) 8

9. A jar has 12 jellybeans, 8 gum balls, and 4 sweet tarts. If Kaleo selects two objects (without replacement) out of the jar, what is the probability that the objects will be different?

- a)  $25/69$
- b)  $44/69$
- c)  $1/18$
- d)  $7/18$
- e)  $45/73$

10. Amy and Ben play a game where they take turns rolling a pair six-sided dice. The game ends when someone obtains a sum of 5. This person is declared the winner. If Amy rolls first, what is the probability of her winning?

- a)  $1/9$
- b)  $8/81$
- c)  $1/2$
- d)  $8/17$
- e)  $9/17$

11. Let  $x$  be a randomly selected number between 1 and 30, inclusive. What is the probability that  $x!$  will have a factor of 100?

- a)  $1/5$       b)  $1/2$       c)  $2/3$       d)  $7/10$       e)  $11/15$

12. Find the probability that the remainder of  $x^2$  divided by 6 will be greater than the remainder of  $x^2$  divided by 4 if  $x$  is a randomly selected integer.

- a)  $1/6$       b)  $1/3$       c)  $1/2$       d)  $2/3$       e)  $5/6$

13. The chance of a student being accepted at BYU Hawai'i, Georgia Tech, and Stanford are 40%, 30%, and 20%, respectively. If a student applies to these three schools, what is the probability of the student being accepted to at least one of these schools?

- a)  $3/20$       b)  $83/125$       c)  $697/1000$       d)  $6/25$       e)  $9/10$

14. If  $P(A) = 1/5$ ,  $P(B) = 3/4$ , and  $P(A \cap B) = 1/10$ , what is the probability that neither A nor B will take place?

- a)  $3/20$       b)  $1/4$       c)  $4/5$       d)  $1/10$       e)  $1/20$

15. In the 1996 Summer Olympics, the probability that the USA will gain the most medals is 26%. Meanwhile, the odds that United Germany will claim the most medals is 4:21. What are the odds of a country other than the USA or Germany of claiming the most medals?

- a) 1:2      b) 9:11      c) 29:21      d) 21:50      e) 11:20

16. How many ways can Pono, Kimo, Hiapo, Leo, and Kekoa stand in a line if Leo and Kekoa refuse to line up next to each other?

- a) 48      b) 60      c) 72      d) 96      e) 120

17. The probability that a boy will enjoy playing tennis is  $\frac{2}{3}$ , while that for a girl is  $\frac{1}{3}$ . If  $P(\text{girl}) = P(\text{boy}) = \frac{1}{2}$ , what is the probability that exactly one of a family's two children will enjoy playing tennis?

- a)  $\frac{1}{3}$
- b)  $\frac{1}{2}$
- c)  $\frac{2}{3}$
- d)  $\frac{5}{9}$
- e)  $\frac{7}{18}$

18. How many distinct ways can one arrange the letters HAWAII?

- a) 30
- b) 120
- c) 180
- d) 360
- e) 720

19. Three different alien races each decide to randomly land on one of Hawaii's four islands (Maui, Kauai, Molokai, and Lanai). What is the probability that exactly two of them will land on the same island?

- a)  $\frac{9}{16}$
- b)  $\frac{16}{25}$
- c)  $\frac{3}{7}$
- d)  $\frac{3}{8}$
- e)  $\frac{3}{10}$

20. In the following set of positive integers, if the median of the numbers is the average of the mean and the unique mode of the numbers, find the product of  $x$  and the median.

DATA: 22, 3, 9, 7, 11, 17, 7, 4,  $x$

- a) 119
- b) 133
- c) 153
- d) 171
- e) 245

21. If a point is randomly selected on a square with side 3 inches, what is the probability that it will be within 1 inch of a corner of the square?

- a)  $\frac{4}{9}$
- b)  $\frac{\pi}{9}$
- c)  $\frac{\pi}{3}$
- d)  $\frac{\pi}{5}$
- e)  $\frac{2}{\pi}$

22. How many distinct ways can 10 different beads be arranged on a necklace?

- a) 45      b) 55      c) 181440      d) 362880      e) 1814400

23. In a cube with side  $x$ , spheres with radii  $x/2$  are centered at each of the vertices. If a point inside the cube is selected at random, what is the probability that it will be within one of the spheres?

- a)  $\pi/6$       b)  $3/4\pi$       c)  $2/3$       d)  $3/\pi$       e)  $2/\pi$

24. How many terms are in the expansion of  $(a+b+c+d)^{12}$ ?

- a) 132      b) 144      c) 178      d) 455      e) 4096

25. If  $x$ ,  $y$ , and  $z$  are 3 randomly chosen integers between 1 and 6, inclusive, what is the probability that they can be the lengths of diagonals of 3 mutually adjacent faces of a rectangular box?

- a)  $25/216$       b)  $28/216$       c)  $29/216$   
 d)  $31/216$       e)  $35/216$

26. In bag A, there are four balls labeled 1, 9, 9, 3. In bag B, there are 4 balls labeled 1, 5, 7, 9. If a person randomly selects a ball from bag A, drops it into bag B, and then selects a ball from bag B, what is the probability that he will have drawn a 9 from bag B?

- a)  $1/4$       b)  $3/10$       c)  $2/5$       d)  $1/2$       e)  $3/5$

27. If a number is randomly selected between  $39916802 (= 11! + 2)$  and  $39916811$ , inclusive, what are the odds of it being composite?

- a) 1:1      b) 2:3      c) 1:9      d) 9:1      e) 1:0

28. In a best-of-5 NBA 1st round playoff game (1st team to 3 victories wins the series), the odds of the Chicago Bulls defeating the Atlanta Hawks in each game are 3:2. What is the probability that someone will win the series in 4 games?

- a) 96/625
- b) 162/625
- c) 216/625
- d) 234/625
- e) 312/625

29. A company assigns 4-digit extensions which are completely random. What is the probability that a worker will receive a 4-digit code that will have repeated digits? (for example 2446 and 1111 are two codes that have repeated digits)

- a) 3/17
- b) 11/25
- c) 83/250
- d) 167/250
- e) 62/125

30. Professor Smith forgot to record a couple of her student's scores (scores are integers ranging from 0 to 100) in her class roll. She, however, remembered that:

- i) Kehau had the highest average
- ii) the lowest test score was 70
- iii) on each test, the three test scores formed arithmetic progressions.

Find the sum of Kehau's 4 tests.

Class Roll

	Kehau	Nalani	Ekela
Test 1	???	100	???
Test 2	87	78	???
Test 3	92	83	???
Test 4	???	82	71

- a) 324
- b) 333
- c) 342
- d) 343
- e) 357