

Mu Alpha Theta National Convention: Denver, 2001
Number Theory Topic Test – Alpha Division

1. For how many ordered pairs of integers (m,n) does m multiplied by n equal 120?
(A) 8 (B) 16 (C) 24 (D) 32 (E) NOTA
2. Find the sum of the positive proper integral factors of 512.
(A) 256 (B) 511 (C) 512 (D) 1,023 (E) NOTA
3. In how many consecutive zeros does the number $634!$ end?
(A) 125 (B) 131 (C) 156 (D) 157 (E) NOTA
4. What is the smallest counting number with exactly 32 positive integer factors?
(A) 720 (B) 800 (C) 840 (D) 2,310 (E) NOTA
5. Find the sum of the smallest pair of positive twin primes whose product is greater than 6,000.
(A) 144 (B) 204 (C) 216 (D) 236 (E) NOTA
6. Find the largest integer d for which there are no nonnegative integer solutions (a, b, c) which satisfy the equation
$$5a + 7b + 11c = d$$

(A) 8 (B) 9 (C) 13 (D) 17 (E) NOTA
7. What is the sum of the positive integral prime divisors of 4,199?
(A) 23 (B) 32 (C) 49 (D) 55 (E) NOTA
8. If 432_8 is equal to 162_n , what is n ?
(A) 14 (B) 15 (C) 21 (D) 23 (E) NOTA
9. Which of the following numbers is congruent to 4 (mod 7)?
(A) 31 (B) 331 (C) 3,331 (D) 33,331 (E) NOTA

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10. Three three-digit integers each have a remainder of 3 when divided by 13. What is the remainder when the product of those numbers is divided by 13?
- (A) 1 (B) 2 (C) 3 (D) 6 (E) NOTA
11. What is the smallest positive integer with exactly 48 positive integer factors?
- (A) 2,520 (B) 3,360 (C) 3,600 (D) 6,930 (E) NOTA
12. When the digits of a positive two-digit integer are reversed, the resulting number is 18 more than the original number. Find the difference when the tens digit of the original number is subtracted from the units digit of the original number.
- (A) 1 (B) 2
(C) 3 (D) no unique solution (E) NOTA
13. Which of the following numbers is equal to 144_6 ?
- (A) 100_8 (B) 220_5 (C) 1010101_2 (D) 36_9 (E) NOTA
14. What is the sum of the positive integer factors of 324?
- (A) 567 (B) 630 (C) 756 (D) 847 (E) NOTA
15. If N is a positive integer and $N \equiv 2 \pmod{3}$ and $N \equiv 3 \pmod{4}$, what is the remainder when N is divided by 6?
- (A) 1 (B) 2 (C) 3 (D) 5 (E) NOTA
16. The sum of the first n counting numbers is equal to S where S is a multiple of 249. What is the smallest possible value for n ?
- (A) 82 (B) 83 (C) 165 (D) 249 (E) NOTA
17. My father is less than forty years older than I am, but more than twenty years older than I am. The product of our ages is 1,288. How old is my father?
- (A) 46 (B) 48 (C) 56 (D) 62 (E) NOTA

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18. A set consists of three positive integers the product of which is 240. What is the smallest possible sum of these integers, if each of the three is relatively prime to each of the other two?
- (A) 19 (B) 24 (C) 32 (D) 45 (E) NOTA
19. If $60a \equiv 1 \pmod{7}$, what is $220a$ congruent to $\pmod{7}$?
- (A) 6 (B) 5 (C) 4 (D) 3 (E) NOTA
20. The 4-digit number $4A4B$ is divisible by 72. What digit does A represent?
- (A) 2 (B) 4 (C) 1 (D) 8 (E) NOTA
21. Katie Holmes shares the same birthday as the oldest living pin-striped fuzzy tortoise, Bart (who is between 20 and 100 years older than Katie). For five consecutive birthdays, Bart was an integer number of times as old as Katie. What is the youngest Bart could have been on the fifth of those birthdays?
- (A) 25 (B) 45 (C) 65 (D) 85 (E) NOTA
22. If $17x \equiv 177 \pmod{1777}$, and $177x \equiv n \pmod{1777}$, which of the following could be n ?
- (A) 7 (B) 484 (C) 711 (D) 324 (E) NOTA
23. What is the smallest possible positive difference between two integers whose product is 1,353?
- (A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA
24. What is the base 10 representation of the base 6 number $\bar{.1}$?
- (A) .2 (B) $.\bar{16}$ (C) $\bar{.3}$ (D) .3 (E) NOTA
25. What is the sum of the 40 smallest positive perfect cubes?
- (A) 640,000 (B) 672,400 (C) 707,281 (D) 885,600 (E) NOTA
26. Find the sum of the 20 smallest natural numbers which each have an even number of positive proper integral factors.
- (A) 270 (B) 8,410 (C) 2,870 (D) 44,100 (E) NOTA

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27. An integer, N , expressed in base B is 6789_B . If N is a multiple of $(B-1)$ and B is less than 20, which of the following could be B ?
- (A) 11 (B) 13 (C) 15 (D) 18 (E) NOTA
28. In the Battle of the Tentacled Eye, 40 humans killed 2,529 aliens. Each human killed a number of aliens which is divisible by 9. One of the humans, Pat B., killed more aliens than any of the other humans. What is the smallest number of aliens that Pat could have killed?
- (A) 54 (B) 63 (C) 72 (D) 81 (E) NOTA
29. What is the hundreds digit of 7^{707} ?
- (A) 3 (B) 5 (C) 7 (D) 9 (E) NOTA
30. If $x \equiv 3 \pmod{41}$ and $p \equiv 6 \pmod{40}$, what is the remainder when x^p is divided by 41?
- (A) 9 (B) 40 (C) 1 (D) 32 (E) NOTA
31. What is the sum of the first 25 perfect squares whose square roots are not perfect squares?
- (A) 979 (B) 5,525 (C) 8,476 (D) 9,455 (E) NOTA
32. What is the third smallest positive integer, x , such that $x \equiv 2 \pmod{4}$, $x \equiv 3 \pmod{9}$, and $x \equiv 7 \pmod{49}$?
- (A) 5,250 (B) 7,014 (C) 7,723 (D) 14,070 (E) NOTA
33. Find the product of the positive integral divisors of 360.
- (A) 360^6 (B) 360^{12} (C) 360^{24} (D) 360^{48} (E) NOTA
34. On the television station MTV, $A\%$ of the airtime is devoted to the airing of quality music, $B\%$ to the airing of other music, $C\%$ to the airing of advertisements, and $D\%$ to the airing of other programming. A , B , C , and D are positive integers and add up to 100. $A < B$, C is four times B , and C is equal to D . If D is a multiple of 11, find A .
- (A) 0 (B) 1 (C) 12 (D) 23 (E) NOTA

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35. What is the remainder when 5^{301} is divided by 13?
(A) 1 (B) 5 (C) 8 (D) 12 (E) NOTA
36. What is the third smallest positive integer that is a multiple of 5 and has no digit greater than 1 when expressed in base 6?
(A) 8,035 (B) 9,115 (C) 9,295 (D) 9,325 (E) NOTA
37. Given that N is a positive integer less than one hundred, if $3^N \equiv N \pmod{100}$, what is N ?
(A) 7 (B) 41 (C) 49 (D) 87 (E) NOTA
38. For how many values of b , $0 < b < 24$, could $p^2 \equiv b \pmod{24}$ for some prime number p ?
(A) 3 (B) 4 (C) 6 (D) 23 (E) NOTA
39. Let $F_{n+1} = F_n + F_{n-1}$ and let $F_1 = F_2 = 1$. Find the smallest positive integer, m , such that $F_{n+m} \equiv F_n \pmod{7}$ for all integers, n .
(A) 8 (B) 12 (C) 16 (D) 24 (E) NOTA
40. $A = \sum_{q=1}^{108} (q!)$ What is the remainder when A is divided by 108?
(A) 0 (B) 9 (C) 45 (D) 81 (E) NOTA