1999 MAΘ National Convention Number Theory — Alpha Division

1. How many four digit numbers without repeated digits can be formed from the numbers 0,2,4,6,8?

A. 24 B. 48 C. 96 D. 116 E. 120

2. How many reversal addition steps does it take to change 96 into a palindrome? (Example, 47 takes one reversal addition step because 47 + 74 = 121 which is a palindrome.)

A. 1 B. 2 C. 4 D. 6 E. 8

3. If m = the number of positive integral divisors of 2700, find the square root of one-fourth m.

A. 2 B. 3 C. 4 D. 5 E. 6

4. What is the remainder when 1999^{1999} is divided by 9?

A. 0 B. 1 C. 2 D. 3 E. 4

5. If 2m - 3 is divisible by 8, which of the following is also divisible by 8?

A. 2m + 3 B. 2m - 8 C. 4m + 8 D. 5m + 1 E. 6m - 9

6. If $A = \sum_{k=3}^{45} k!$, then what is the remainder when A is divided by 240?

A. 144 B. 150 C. 152 D. 170 E. 220

7. Which one of the following is not divisible by 7, 9, or 11?

A. 140 B. 232 C. 319 D. 360 E. 1078

8. If n = the sum of the positive integral divisors of 5400, find the largest prime factor of n.

A. 2 B. 3 C. 18 D. 31 E. 48

9. In a certain base, b, 2 times the number represented by the numeral 137 is 5 times the number represented by the numeral 51. Write 25_{10} in base b.

A. 7 B. 9 C. 23 D. 27 E. 46

10. How many 4 digit numbers have at least one 2 or one 3?

A. 3584 B. 4096 C. 5416 D. 6416 E. 7416

11. A child's stacking toy is made of 10 different plastic cubes with side lengths 1 cm, 2 cm, 3cm, . . . 10 cm. Instead of stacking them, Bobby decides to pour his grape juice in them. To the nearest tenth, how many 2 liters of grape juice will the toys hold?

A. 1.5 B. 1.7 C. 2.0 D. 2.2 E. 2.7

12. If m and n are a set of twin primes such that 18 more than twice m is equal to 7 less than 3 times n, find $m^2 - n^2$.

A. 37 B. 72 C. 100 D. 120 E. 220

13. Suppose $(A \cap B) \cup (A \cap C) = D$, then find $B \cup (A \cap C)$.

A. $A \cup D$ B. $B \cup C$ C. $D \cup B$ D. $D \cap B$ E. $\varnothing$

14. When simplified completely, , $\frac{1}{(2!)(4!)} + \frac{1}{(3!)(3!)} = \frac{m}{n}$,, where m and n are integers and relatively prime, find the least common multiple of m and n.

A. 24 B. 144 C. 151 D. 1008 E. 2016

15. How many prime numbers between 50 and 100 have remainder 3 when divided by 6?

A. 0 B. 2 C. 5 D. 8 E. 11

16. For all integral values of m, $m^2 + a$ is not divisible by 10. If a is a single-digit positive number, find the sum of all possible values of a.

A. 2 B. 5 C. 12 D. 15 E. 20

17. What is the remainder when $(19^{19} + 99^{99})$ is divided by 7?

A. 1 B. 2 C. 4 D. 5 E. 6

18. How many pairs of positive integral solutions does the equation $2x + 11y = 35$ have?

A. none B. 2 C. 3 D. 5 E. 6

19. Which of the following has the same remainder when divided by 4, 5, and 9?

A. 253 B. 287 C. 321 D. 357 E. 722

20. Using the digits 4, 6, 7, and 9, only once each, how many four digit numbers can be formed which are divisible by 11?

A. 4 B. 8 C. 12 D. 24 E. 256

21. Let m = the largest integral divisor of k^3-k where k is any odd integer. Let n= the largest integral divisor of p^3-p where p is any even integer. Find $\left|\frac{m-n}{2}\right|$.

A. 1 B. 1.5 C. 6 D. 9 E. 18

22. Suppose m is an integer such that $m(9!) = \frac{151!}{142!}$. Find the largest prime divisor of m.

A. 5 B. 7 C. 13 D. 29 E. none of these

23. Which of the following is divisible by 3 for all integral values of n?

A. n^3-7n+3 B. n^3-8n+3 C. n^2-1 D. n^3-2n E. n^3 -3n

24. Find the next term in the sequence: 3, 5, 9, 17, 32, 58, . . .

A. 48 B. 84 C. 95 D. 100 E. 116

25. Simplify completely: $[\{A\cap(B\cup C)\}\cap\{A\cup(B\cap C)\}]\cap(C\cup B^C)$

A. $A\cap B\cap C$ B. $A\cup B$ C. $A\cap B$ D. $A\cap C$ E. $\varnothing$

26. . How many zeros are at the end of $(22!)^2$ when it's written in base 4?

A. 4 B. 8 C. 19 D. 22 E. 38

27. If you choose a number from the set 11-30 inclusive, and x is the probability you've chosen a deficient number, which of the following describes x?

A. $x<0.2$ B. $0.2\leq x<0.4$ C. $0.4\leq x<0.55$ D. $0.55\leq x<0.7$ E. $0.7\leq x<0.85$

28. If $25\equiv 85 \bmod m$ and $52\equiv 92 \bmod m$, then m must divide which of the following?

A. 2 B. 5 C. 10 D. 20 E. 25

29. A triangular pyramid of ping pong balls is being formed so that each ball rests in the space of the 3 below it. Only the bottom n levels have been completed, using 185 balls. Find n.

A. 3 B. 4 C. 5 D. 6 E. 7

30. How many integral pairs (x,y) are solutions to the following system of equations:
$(x-1)^2 - (y-3)^2 = 0$ and $\left|\frac{1}{3}x + y\right| < 3$.

A. 0 B. 11 C. 13 D. 17 E. ∞ many

Tie Breaker 1: How many of the following numbers form English words when converted from decimal to hexadecimal?

189, 3245, 4077, 57069

Tie Breaker 2: If 792 divides the integer 13xy45z, find the digits x, y, and z.

Tie Breaker 3: How many Pythagorean triples of the form $m^2 - n^2$, $2mn$, $m^2 + n^2$ have all three integers between 100 and 200 and include at least one multiple of 7?