

Mu Alpha Theta National Convention: Seattle, 1997
Alpha Number Theory Topic Test

1. How many proper factors does 15424 have?
(A) 11 (B) 12 (C) 13 (D) 14 (E) NOTA
2. In how many zeros does 4715! end?
(A) 873 (B) 912 (C) 1043 (D) 1176 (E) NOTA
3. What is the sum of the LCM and GCF of 36, 144, and 78?
(A) 1878 (B) 1922 (C) 1964 (D) 1992 (E) NOTA
4. What is the greatest prime less than 457?
(A) 451 (B) 449 (C) 443 (D) 439 (E) NOTA
5. Determine the sum of the cubes of all the natural numbers up to n.
(A) $\frac{n^2(n+1)^2}{4}$ (B) $\frac{2n(n+1)(2n+1)}{12}$
(C) $\frac{3n(n+1)}{6}$ (D) $\frac{2n(n+1)(n+2)^2(n+3)}{3}$ (E) NOTA
6. Evaluate: $\sum_{i=1}^{49} \binom{49}{i}$
(A) 2^{50} (B) 2^{49} (C) $2^{50} - 2^0$ (D) $2^{49} - 2^0$ (E) NOTA
7. How many prime numbers are factors of 55! ?
(A) 15 (B) 16 (C) 17 (D) 18 (E) NOTA
8. For each n in the set of natural numbers from 1 to 18, the remainder when t is divided by n is (n-1). What is the smallest possible value for t?
(A) 12,252,239 (B) 1,533,471 (C) 1,327,553 (D) 911,279 (E) NOTA
9. A three digit number is 495 greater than the number formed when its digits are reversed. What is the second largest such number?
(A) 883 (B) 984 (C) 893 (D) 873 (E) NOTA

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10. $A = \sum_{q=1}^{32} q!$ What is the remainder when A is divided by 72?
(A) 9 (B) 10 (C) 11 (D) 12 (E) NOTA
11. The number 5A427B is divisible by 12. How many possible values are there for A+B?
(A) 3 (B) 5 (C) 6 (D) 4 (E) NOTA
12. Evaluate: $\sqrt{9 + \sqrt{9 + \sqrt{9 + \dots}}}$
(A) $\frac{1 + \sqrt{37}}{2}$ (B) $1 + \sqrt{5}$ (C) $\frac{3 + 3\sqrt{3}}{2}$ (D) $\frac{7}{2}$ (E) NOTA
13. What is the product of 1234_9 and 432_9 , expressed in base 9?
(A) 500028_9 (B) 523178_9 (C) 545408_9 (D) 564318_9 (E) NOTA
14. What is the sum of the positive divisors of 6561?
(A) 8438 (B) 8542 (C) 9213 (D) 9841 (E) NOTA
15. Which of the following numbers is both triangular and perfect?
(A) 465 (B) 487 (C) 496 (D) 528 (E) NOTA
16. A store sold 72 pencils for \$4AB.64. How many possible pencil prices are there? (Pencils cost an integer number of cents.)
(A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA
17. How many Pythagorean triangles with perimeters less than 100 have side lengths which are arithmetic sequences?
(A) 8 (B) 11 (C) 13 (D) 14 (E) NOTA
18. What is the units digit of 17^{171} ?
(A) 3 (B) 9 (C) 7 (D) 1 (E) NOTA

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19. $d = 7 \pmod{12}$. Which of the following could d equal in $\pmod{30}$?
- (A) 17 (B) 12 (C) 30 (D) 25 (E) NOTA
20. A number m , when written in base $c > 11$, is equal to AB , where A is $c-4$ and B is 6. What is m , expressed in base $(c-2)$?
- (A) 102 (B) 87 (C) 54 (D) 215 (E) NOTA
21. If $x = a \pmod{37}$ is a solution of the equation $9x = 3 \pmod{37}$, determine the sum of the possible values of a .
- (A) 25 (B) 32 (C) 39 (D) 43 (E) NOTA
22. A man performs a multiplication problem, then decides to replace any prime digit with the letter p . Curiously enough, every digit was prime, resulting in the following problem. What is the product?
- $$\begin{array}{r}
 p \ p \ p \\
 \underline{\quad p \ p} \\
 p \ p \ p \ p \\
 \underline{p \ p \ p \ p} \\
 p \ p \ p \ p \ p
 \end{array}$$
- (A) 33572 (B) 27353 (C) 32577 (D) 25575 (E) NOTA
23. In how many zeros does $148!$ end when expressed in base 12?
- (A) 70 (B) 71 (C) 72 (D) 73 (E) NOTA
24. Find the sum of the three smallest natural numbers with exactly 8 factors.
- (A) 72 (B) 94 (C) 118 (D) 124 (E) NOTA
25. Express 888_9 in base 18.
- (A) 142 (B) 248 (C) 222 (D) 482 (E) NOTA
26. In how many bases, $b > 8$, will 35_b and 58_b not be relatively prime?
- (A) 4 (B) 7 (C) 9 (D) 12 (E) NOTA

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27. In what base, $b > 9$, is 297_b a factor of 792_b ?
- (A) 11 (B) 12 (C) 13 (D) 14 (E) NOTA
28. A good-natured jailer decides to release some prisoners. He walks through the cell block containing n cells, and first toggles the lock state on every door, causing all the doors to be unlocked. Then he walks through again, toggling the state of every second door, causing all the even numbered cells to be locked again. He repeats this process n times (the last time, he only toggles the state of the n th cell). Which of the following cell numbers had lucky occupants at the end of this process?
- (A) 17160 (B) 17161 (C) 17162 (D) 17163 (E) NOTA
29. Find the sum of all values of $b > 1$ for which 11111_b is a perfect square?
- (A) 3 (B) 7 (C) 12 (D) 18 (E) NOTA
30. The product of three integers in arithmetic progression is of the form $10^{*****}2$, where each $*$ represents a digit, though not necessarily the same digit for each $*$. What is the largest of the three integers?
- (A) (B) 248 (C) 222 (D) 482 (E) NOTA