6

Number Theory Topic Test Solutions

① just check

$n=1$ 2 $n=5$ 121

$n=2$ 3 $n=6$ 721

$n=3$ 7 $n=7$ 5041

$n=4$ 25

There are 3.

ans. C

② $\binom{10}{0}+\binom{10}{1}+\dots+\binom{10}{10}=2^{10}=1024$ ans. D

③ Twin primes differ by 2. Twin primes are 2 consecutive odd primes.

List them: 3 ∧ 5, 5 ∧ 7, 11 ∧ 13, 17 ∧ 19, 29 ∧ 31, 41 ∧ 43

ans. C

④ Many will recognize formula. Can check if don't recognize.

E is not true. $1+8+27 \neq \frac{3(3+1)^2}{4}$

ans E

⑤ Set up equation: $10x + 25y = 555$ ans B

$5y \equiv 5 \mod 10$

$y \equiv 1$ $x = 53$ $S = 54$

$y \equiv 11$ $x = 29$ $S = 40$

$y \equiv 21$ $x = 3$ $S = 24$

$y \equiv 31$ —

⑥ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

18 19 20 21 22 23 24 25 26 27 28 29 30

31 32 33 34 35 36 37 38 39 40 41 42 43

44 45 46 47 48 49 50

count 15 This is all primes < 50.

ans C

⑦ $f(n) = n(n+1) + 17$ ans D

by inspection $n = 16$

$16(17) + 17 =$

$(17)(16+1) =$

$(17)(17)$ composite

(8) Factor

$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

$360 = 2^3 \cdot 3^2 \cdot 5$ ans D

$\therefore$ No $= 4 \cdot 3 \cdot 2 = 24$

(9) $10 = 2 \cdot 5$. always "enough" 2's. ans A
So count # of 5's

$1991 \div 5 = 398$
$1991 \div 25 = 79$
$1991 \div 125 = 15$
$1991 \div 625 = 3$
$\overline{495}$

(10) $2 \cdot 3 \cdot 4 \cdot 5$ Ans B
L.C.M $2^2 \cdot 3 \cdot 5 = 60$

(11) $\dfrac{10t+d}{t+d} = 4$ ans D

$10t+d = 4t+4d$
$d = 2t$

$t=1$ $t=2$
$d=2$ $d=4$
$\frac{12}{3}$ $\frac{24}{6} = 4$

(12) $b^2+d+b^2+b^2-d = ?$ ans D
$3b^2 = ?$

$b^2 = \frac{\quad}{3}$

The only sum which gives a square is 75.
The sequence $49 > 25 > 1$ but not necessary to find.

(13) $1 + 1\cdot 2 + 1\cdot 2\cdot 3 + 1\cdot 2\cdot 3\cdot 4 + 1\cdot 2\cdot 3\cdot 4\cdot 5 + \ldots + 91!$

$1 + 2 + 6 + 24 + 15(8 + \ldots)$

$33 + 15(8 + \ldots)$ Ans A

$3 + 15(2 + 8 + \ldots)$

$\therefore$ Remainder is 3

(14)

$$\begin{array}{r|l} 2 & 1571724 \\ 2 & 785862 \\ 3 & 392931 \\ 3 & 130977 \\ 3 & 43659 \\ 3 & 14553 \\ 3 & 4851 \\ 7 & 1617 \\ 7 & 539 \\ 7 & 77 \\ 11 & 11 \\ & 1 \end{array}$$

By inspection

1, 2, 3, 4, 6, 7, 9, 11, 12, 14 Ans D

$\therefore$ 10 numbers

(15) $\lim_{n\to\infty} \dfrac{u_{n+1}}{u_n}$ $u_{n+1} = u_n + u_{n-1}$ def'n E

$\lim_{n\to\infty} \dfrac{u_n + u_{n-1}}{u_n}$ $\lim_{n\to\infty}\left(1 + \dfrac{1}{\dfrac{u_{n-1} + u_{n-2}}{u_{n-1}}}\right) =$

$\lim_{n\to\infty}\left(1 + \dfrac{u_{n-1}}{u_n}\right)$

$\lim_{n\to\infty}\left(1 + \dfrac{1}{\dfrac{u_n}{u_{n-1}}}\right)$ $\lim_{n\to\infty}\left(1 + \dfrac{1}{1 + \dfrac{u_{n-2}}{u_{n-1}}}\right)$

etc.

(16) $AB = (b-2)2_b$

$(b-2)b^1 + 2b^0 =$ ans C

$b^2 - 2b + 2 =$

choose 101 so that the expression will factor

$b^2 - 2b + 2 = 101$

$b^2 - 2b - 99 = 0$

$(b-11)(b+9) = 0$

$b = 11 \quad b \neq -9$

(17) $.222\ldots = \frac{2}{7} + \frac{2}{49} + \frac{2}{343} + \ldots$ ans A

Inf. Geometric Series

$S_n = \frac{\frac{2}{7}}{1-\frac{1}{7}} = \frac{1}{3} = .333\ldots$

$\therefore b = 3$

(18) From #8

$360 = 2^3 \cdot 3^2 \cdot 5^1$

$Sum = \frac{2^4-1}{2-1} \cdot \frac{3^3-1}{3-1} \cdot \frac{5^2-1}{5-1}$

$= 15 \cdot 13 \cdot 6$

$= 15 \cdot 78$

$= 780 + 390$

$= 1170$

(19) Triangular nos. from Pascal's Δ ans. D

1, 3, 6, 10, 15, 21, 28, 36, ...

Perfect numbers

6, 28, 496

Triangular numbers: $\frac{n^2+n}{2}$

(20) Observations

b has to be even

\$ a67.9b must be divisible by 4. $\therefore b = 2$ or 6 — ans B

\$ a67.9b must be divisible by 9

$\therefore$ sum of digits must be a multiple of 9

Try $b = 2 \quad \therefore a = 3$

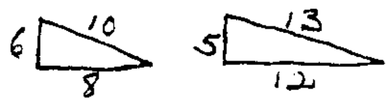
$$72 \overline{)367.92} \quad 5.11 \qquad a + b = 5$$

Actually not necessary to find a and b just use sum of digits must be divisible by 9.

(21) $a + b + c = \frac{1}{2}ab$ — Ans B

By trial and error.

There are 2.



(22) $5^{38} \div 11$. Find remainder.

$5^2 \equiv 3 \mod 11$ — ans B

$5^4 \equiv 9 \mod 11$

$5^8 \equiv 4 \mod 11$

$5^{16} \equiv 5 \mod 11$

$5^{32} \equiv 3 \mod 11$

$$5^{32} \cdot 5^2 \cdot 5^4 \equiv 3 \cdot 3 \cdot 9 \equiv 4 \mod 11$$

(23) 2 is a primitive root of 9

Formula

$r_1 = r + p$ where r is a given primitive root of mod p^2

since $r = 2$ is given and $p^2 = 9$ C

we have

$r_1 = 2 + 3 = 5$

(24) $x \equiv a \mod 21$ Ans A.

$6x \equiv 15 \mod 21$

$GCD(6, 21) = 3$

$\therefore 3$ solutions

$6x \equiv 15 \mod 21$

$6x \equiv 36 \mod 21$

$x \equiv 6 \mod 21$

$6x \equiv 78 \mod 21$

$x \equiv 13 \mod 21$

$6x \equiv 120 \mod 21$

$x \equiv 20 \mod 21$

Sum: $6 + 13 + 20 = 39$

25 $$2^{22}+1 = (2^{11}+1)^2 - 2^{12}$$

E

$$= (2^{11}+1+2^{6})(2^{11}+1-6)$$

$$= 2113 \cdot 1985$$

$$\therefore \ 2113$$