5 Solutions for Matrices and Determinants Topic Test
Alpha Level MAΘ 1999

1. 3 rows and 5 columns $\underline{3 \times 5}$ (B)

2. $3[33] - 5[25] = 99 - 125 = \underline{-26}$ (C)

3. $x - 6 = {}^{-}9$ $\quad x = -3$ $\qquad 5 + y = 1$ $\quad y = -4$ $\qquad x + y = -3 + (-4) = \underline{-7}$ (A)

4. $15 - (-11x) = -18 - (-24)$
$15 + 11x = -18 + 24$
$11x = 6 - 15$
$11x = -9$
$x = \underline{\frac{-9}{11}}$ (D)

5. $\frac{1}{2} \left| \begin{vmatrix} -11 & 4 & 1 \\ -5 & -6 & 1 \\ 4 & 7 & 1 \end{vmatrix} \right| = \frac{1}{2} \left| 66 + 16 - 35 - (-24) - (-77) - (-20) \right|$
$\frac{1}{2}|168| = \underline{84}$ (C)

6. $-\frac{1}{4} + 4 + \left(-\frac{1}{5}\right) - \frac{2}{3} - \frac{1}{2} - \frac{3}{5} = \frac{-15 + 240 - 12 - 40 - 30 - 36}{60}$
$= \underline{\frac{107}{60}}$ (A)

7. $3x + 2y = 4$ $\quad 5x + 5y = 5$ $\qquad x + y = 1$ $\quad 3x + 2y = 4$

$$\begin{array}{r} 2x + 2y = 2 \\ 3x + 2y = 4 \\ \hline x = 2 \end{array}$$

$y = -1$ $\qquad x + y$ $\quad 2 + (-1) = \underline{1}$ (C)

(8) All statements are true (D)

I. definition of matrix addition TRUE

II. $A + B = C$ where A, B and C are all $m \times n$ matrices TRUE

III. $A + B = B + A$ for all $m \times n$ matrices TRUE

IV. $[A+B]+C = A+[B+C]$ for all $m \times n$ matrices TRUE

V. def of additive inverse TRUE

(9)

$$A \cdot B = \begin{bmatrix} (-32+28+75) & (18-56-210) \\ (-192+40-20) & (108-80+56) \end{bmatrix} = \begin{bmatrix} 71 & -248 \\ -172 & 84 \end{bmatrix}$$

(B)

(10)

$$[P+Q]^2 = (P+Q)(P+Q) = \boxed{P^2 + PQ + QP + P^2}$$

(E) Since Matrix Multiplication is not commutative, Answer must remain in above form.

(11)

$$5x + 4y = -3$$
$$3x - 5y = -24$$

$\Rightarrow$

$$25x + 20y = -15$$
$$12x - 20y = -96$$
$$37x = -111$$
$$x = -3$$

$$y = \frac{-3 - 5(-3)}{4}$$
$$y = \frac{-3+15}{4} = \frac{12}{4} = 3$$

$$y - x = 3 - (-3) = \underline{6}$$

(D)

(12.)

$$-13440 + 768x + 16200 - 9600 - 756x + 23040 = 16347$$
$$12x + 16200 = 16347$$
$$12x = 147$$
$$x = \frac{147}{12} = \frac{49}{4} \Rightarrow 49 + 4 = 53$$

(C)

(13) $A^{-1} = \frac{1}{10}\begin{bmatrix} 6 & -2 & 2 \\ 4 & 2 & 8 \\ 3 & -1 & 6 \end{bmatrix} \Rightarrow$ Sum of elements =

$A^{-1} = \begin{bmatrix} .6 & -.2 & .2 \\ .4 & .2 & .8 \\ .3 & -.1 & .6 \end{bmatrix} = \boxed{2.8}$ (C)

(14) The rank of a matrix is the order of the highest order non-zero determinant which can be formed by deleting rows or columns. Thus, $\begin{vmatrix} 5 & -2 \\ 0 & -1 \end{vmatrix}$ has an order of $\underline{2}$ and the rank of the matrix is 2. (A)

(15) Cofactor of $a_{21} = (-1)^{2+1}\begin{vmatrix} -15 & 19 \\ 16 & 24 \end{vmatrix} = (-1)(-664) = \underline{664}$ (D)

(16) $a^3 + 9ab - b^3 + 3ab^2 - ab - 3a^2b$

$a^3 - 3a^2b + 3ab^2 - b^3 + 8ab$

$\underline{(a-b)^3 + 8ab}$ (C)

(17) $4x - 12 - 10x \geq 0$

$-6x - 12 \geq 0$ (B)

$-6x \geq 12$

$\underline{x \leq -2}$

(18) Method 1

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & -1 \\ 1 & 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -24 \\ 9 \end{bmatrix}$

Use Calculator

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \\ -5 \end{bmatrix}$

let A =

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} -2 \\ -24 \\ 9 \end{bmatrix}$

(Method 2- Use Cramer's Rule Not shown)

$\underline{z = -5}$ (A)

(19) det of coefficients $\neq 0$ if solutions exist

$$\therefore \begin{vmatrix} 1 & 5 & 1 \\ 2 & 3 & -2 \\ 2 & -1 & k \end{vmatrix} = 0 \Rightarrow 3k - 20 - 2 - 6 - 2 - 10k = 0$$

$$-7k - 30 = 0$$

$$\underline{k = -\frac{30}{7}}$$ (A)

(20) $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Image under R_{135° $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 135 & -\sin 135 \\ \sin 135 & \cos 135 \end{bmatrix} \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \end{bmatrix}$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$ (D)

(21) det of Inverse A $= \frac{1}{\det A} = \frac{1}{(-300+630+2552+495-320-3045)}$

$$= \frac{1}{12} = \frac{A}{B} \qquad A+B = 1+12 = \underline{13}$$

(D)

(22) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$

$2x + y = \lambda x \Rightarrow y = \lambda x - 2x$

Substitute into 2nd equation

$x + 2y = \lambda y$

$x + 2(\lambda x - 2x) = \lambda(\lambda x - 2x)$

$x + 2\lambda x - 4x = \lambda^2 x - 2\lambda x$

$\lambda^2 x - 4\lambda x + 3x = 0$

$x(\lambda^2 - 4\lambda + 3) = 0$

$x(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda = 3 \quad \lambda = 1}$ (B)

(23) $\frac{1}{6}\left|\begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 0 & 2 & 1 \\ 3 & -1 & 1 & 1 \\ 0 & 1 & 3 & 1 \end{vmatrix}\right| = \frac{1}{6}\left|-6\right| = \frac{1}{6}\cdot 6 = \underline{1}$

(B)

use Calculator to evaluate det

(24) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ by Switching Columns and rows, the same matrix results.

(B)

(25) A set of homogenous equations must be in the form of $ax + by + cz = 0$ where (x, y, z) are the variables in some form such as x_1, x_2, x_3.

(A)

(26) The trace of matrix is the sum of the elements in the principal diagonal.

$\therefore\ 5+2+4+3 = \underline{14}$ (C)

(27) An adjoint of a matrix is the transpose of the cofactors of the matrix.

Adjoint of A is $\begin{bmatrix} -1 & 8 & -5 \\ 3 & 13 & 15 \\ 10 & -6 & 13 \end{bmatrix}$ Element in the 1st row 3rd Column is $\underline{-5}$ (B)

(28) $z = \dfrac{\begin{vmatrix} -3 & 4 & 3 \\ 1 & 2 & 9 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} -3 & 4 & -1 \\ 1 & 2 & -3 \\ 0 & 1 & -5 \end{vmatrix}} = \dfrac{6+0+3-0+27+4}{30+0-1+0-9+20} = \dfrac{40}{40} = \underline{1}$

(C)

(29) $\cos A \cos B - \sin A \sin B = \cos(A+B)$ (A)

(30) D By definition, a Jordon matrix has the diagonal element all equal (and non-zero) and whose elements above the principal diagonal are equal to 1, but all other elements are 0

Matrices and Determinants Tiebreakers

1. Let $A = \begin{bmatrix} 5 & 4 \\ 7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}$ Find the largest element in $(AB^{-1} + BA^T)$.

Answer: 15

Solution: $A = \begin{bmatrix} 5 & 4 \\ 7 & 6 \end{bmatrix}$; $A^T = \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$; $B = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}$; $B^{-1} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$

$$(AB^{-1} + BA^T) = \begin{bmatrix} 5 & 4 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 9 & -13 \\ 13 & -19 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 15 & -5 \\ 14 & -18 \end{bmatrix}$$

15 is the largest element

2. Find the smallest real value of K so that the system of equations below has no solution.

$$\begin{aligned} 2x - 3y + z &= 1 \\ x - y + 2z &= 0 \\ x - 3y + Kz &= 2 \end{aligned}$$

Answer: K = - 4

Solution: $\begin{vmatrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 1 & -3 & K \end{vmatrix} = 0$; $(-2K - 6 - 3) - (-1 - 12 - 3K) = 0$; $K = -4$

3. Le t S be the set of all 2x2 matrices. How many of the following properties are exhibited by all the members of S with regard to multiplication?

I. closure
II. Commutativity
III. All members have inverses
IV. There is an identity matrix

Answer: 2
Solution: Properties exhibited are closure and identity. Matrix multiplication is not commutative and a matrix with determinant of 0 does not have an inverse.