

Mu Alpha Theta National Convention: Seattle, 1997  
Alpha Matrices and Vectors Topic Test

1. Evaluate:  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 3 \\ 7 & -9 & 4 \\ 1 & 1 & -6 \end{bmatrix}$

(A)  $\begin{bmatrix} 3 & 4 & 6 \\ 11 & 14 & 10 \\ 6 & 7 & 15 \end{bmatrix}$

(B)  $\begin{bmatrix} 4 & 2 & -7 \\ 3 & 5 & -2 \\ 12 & -7 & 3 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & 2 & 0 \\ -3 & 4 & 2 \\ -6 & -7 & 15 \end{bmatrix}$

(D)  $\begin{bmatrix} 5 & 0 & 6 \\ 11 & -4 & 10 \\ 8 & 9 & 3 \end{bmatrix}$

(E) NOTA

2. Evaluate:  $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 7 & -3 \\ 1 & 4 \end{bmatrix}$

(A)  $\begin{bmatrix} 20 & -12 \\ 4 & 7 \end{bmatrix}$

(B)  $\begin{bmatrix} 28 & -9 \\ 2 & 4 \end{bmatrix}$

(C)  $\begin{bmatrix} 25 & -24 \\ 13 & -10 \end{bmatrix}$

(D)  $\begin{bmatrix} 31 & 0 \\ 15 & -2 \end{bmatrix}$

(E) NOTA

3. Evaluate:  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$

(A)  $\begin{bmatrix} -1 & -5 \\ 5 & -2 \end{bmatrix}$

(B)  $\begin{bmatrix} 6 & 9 \\ 1 & -1 \end{bmatrix}$

(C)  $\begin{bmatrix} 5 & 11 & -7 \\ 4 & -3 & -18 \end{bmatrix}$

(D)  $\begin{bmatrix} 7 & -2 & 3 \\ 0 & 1 & 12 \end{bmatrix}$

(E) NOTA

4. Evaluate:  $\begin{vmatrix} 4 & -2 \\ 1 & 8 \end{vmatrix}$

(A) 34

(B) 30

(C) 28

(D) 16

(E) NOTA

5. Evaluate:  $\begin{vmatrix} 5 & -4 & 3 \\ 2 & -1 & -6 \\ 7 & 8 & 9 \end{vmatrix}$

(A) 494

(B) 317

(C) 347

(D) 219

(E) NOTA

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6. Evaluate:  $\begin{bmatrix} 5 & 3 \\ -2 & -7 \end{bmatrix}^{-1}$

- (A)  $\begin{bmatrix} 3 & -2 \\ -4 & \frac{1}{2} \end{bmatrix}$  (B)  $\begin{bmatrix} \frac{5}{2} & 4 \\ -2 & -3 \end{bmatrix}$  (C)  $\begin{bmatrix} -3 & -4 \\ 2 & \frac{5}{2} \end{bmatrix}$  (D)  $\begin{bmatrix} \frac{1}{5} & 4 \\ -2 & -\frac{2}{3} \end{bmatrix}$  (E) NOTA

7. What is the trace of  $\begin{bmatrix} 1 & 2 & 6 & -4 \\ 5 & -8 & 7 & 7 \\ -9 & 10 & -14 & 12 \\ 13 & 10 & -15 & 16 \end{bmatrix}$ ?

- (A) 7 (B) 1 (C) -2 (D) -5 (E) NOTA

8. The system of equations

$$4x - 3y + z = 17$$

$$2x - y + 3z = -12$$

$$8x + Ay - 3z = 11$$

has a unique solution. Which value of A is not possible?

- (A) 11 (B) -3 (C) -7 (D) 6 (E) NOTA

9. Every point in the Cartesian plane is subjected to the transformation  $\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$ . Under this transformation, a square region becomes:

- (A) A circular region with area equal to that of the square (B) A square with twice the area of the original.  
(C) A square with four times the area of the original (D) A non-rectangular quadrilateral. (E) NOTA

10. Evaluate:  $\begin{bmatrix} \cos 20^\circ & \sin 20^\circ \\ -\sin 20^\circ & \cos 20^\circ \end{bmatrix}^6$

- (A)  $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$  (B)  $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$  (C)  $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$  (D)  $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$  (E) NOTA

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11. A matrix  $M$  is reduced by a series of elementary row operations to the matrix  $4I$ . What matrix will the same sequence of row operations applied to  $8I$  will yield?
- (A)  $32M$       (B)  $32M^{-1}$       (C)  $2M$       (D)  $2M^{-1}$       (E) NOTA
12. Vector  $A$  points from the point  $(6, -1, 7)$  to the point  $(-3, 2, 4)$ . What is the vector parallel to vector  $A$  and pointing to the point  $(-2, -3, -8)$ ?
- (A)  $[9, -3, 3]$       (B)  $[-9, 3, -3]$       (C)  $[9, 3, 3]$       (D)  $[-9, 3, 3]$       (E) NOTA
13. What is the equation of the plane perpendicular to the vector  $[8, 2, 3]$  and passing through the point  $(7, 12, -5)$ ?
- (A)  $8x + 2y + 3z = 65$       (B)  $-8x + \frac{1}{2}y + 3z = -65$   
(C)  $8x - \frac{1}{2}y + 3z = 35$       (D)  $\frac{1}{8}x + \frac{1}{2}y + \frac{1}{3}z = \frac{125}{24}$       (E) NOTA
14. What is the equation of the plane through the points  $(4, -1, 3)$ ,  $(8, 2, 3)$ , and  $(-2, 5, 5)$ ?
- (A)  $5x + y - 3z = 10$       (B)  $3x + 3y - 2z = 3$   
(C)  $-2x + 4y - z = -15$       (D)  $3x - y - 21z = 79$       (E) NOTA
15. What is the cross product of the vectors  $[4, 7, -3]$  and  $[-2, -2, -7]$ ?
- (A)  $[17, 12, -8]$       (B)  $[43, -22, 6]$       (C)  $[-8, 27, 14]$       (D)  $[19, -4, 61]$       (E) NOTA
16. What is the sum of the eigenvalues of the matrix  $\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$ ?
- (A)  $\frac{7}{3}$       (B) 5      (C)  $\frac{11}{3}$       (D)  $\frac{22}{3}$       (E) NOTA

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17. Which of the following are eigenvectors of the matrix  $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ ?

I.  $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

II.  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

III.  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

IV.  $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

V.  $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

- (A) I & II      (B) III & V      (C) II & IV      (D) I & IV      (E) NOTA

18. What is the volume of the pyramid defined by the points  $(-1, 2, 3)$ ,  $(2, -3, 5)$ ,  $(4, -1, 2)$ , and  $(-3, 6, 3)$ ?

- (A)  $\frac{16}{3}$       (B)  $\frac{31}{6}$       (C) 5      (D)  $\frac{29}{6}$       (E) NOTA

19. Evaluate: 
$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 2 & 3 & 4 & 5 \\ -1 & -2 & 3 & 4 & 5 \\ -1 & -2 & -3 & 4 & 5 \\ -1 & -2 & -3 & -4 & 5 \end{vmatrix}$$

- (A) 1920      (B) 1860      (C) 1790      (D) 1750      (E) NOTA

20. If the determinant of matrix M is -4, what is the determinant of  $M^{-1}$ ?

- (A) .25      (B) 4      (C) -.25      (D) -4      (E) NOTA

21. Which of the following matrices has a determinant different from all the others?

- (A)  $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$       (B)  $\begin{bmatrix} b & c & a \\ e & f & d \\ h & i & g \end{bmatrix}$       (C)  $\begin{bmatrix} c & b & a \\ f & e & d \\ i & h & g \end{bmatrix}$       (D)  $\begin{bmatrix} i & h & g \\ f & e & d \\ c & b & a \end{bmatrix}$       (E) NOTA

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22. Solve the system of equations for  $z$ .

$$w + x + y + z = 12$$

$$2w - x + 3y + z = 9$$

$$-w - 3z + y + 2x = -15$$

$$3w + x - 2y + 2z = 18$$

- (A) 3                      (B) 5                      (C) 7                      (D) 9                      (E) NOTA

23. If  $A$  and  $B$  are 3 by 3 matrices whose  $(i,j)$ th elements are  $ij$  and  $\frac{i}{j}$  respectively, what is the sum of the largest and smallest entries in  $AB$ ?

- (A) 30                      (B) 28                      (C) 36                      (D) 33                      (E) NOTA

24. The transformation matrix  $\begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix}$  is applied to the points of the circle

$$(x + 3)^2 + (y - 2)^2 = 16. \text{ What is the resulting shape?}$$

- (A) A circle              (B) A parabola              (C) A hyperbola              (D) An ellipse              (E) NOTA

25. If  $A$  and  $B$  are  $n$  by  $n$  matrices, which of the following matrices is equal to matrix  $C$ , given

$$\text{that } c_{ij} = \sum_{q=1}^n a_{jq} b_{iq} ?$$

- (A)  $AB$                       (B)  $A^{-1}B$                       (C)  $B'A$                       (D)  $B^{-1}A$                       (E) NOTA

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26. If transformation matrix M is such that  $M \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ w \\ y \\ v \\ x \end{bmatrix}$  for all values of v, w, x, y, and z, then

what is  $M^t \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}$  ?

- (A)  $\begin{bmatrix} v \\ x \\ y \\ w \\ z \end{bmatrix}$       (B)  $\begin{bmatrix} w \\ y \\ v \\ x \\ z \end{bmatrix}$       (C)  $\begin{bmatrix} x \\ v \\ y \\ w \\ z \end{bmatrix}$       (D)  $\begin{bmatrix} y \\ w \\ z \\ x \\ v \end{bmatrix}$       (E) NOTA

27. For what value of a will the vector [5, 1, a] be perpendicular to the vector [3, -1, 2]?

- (A) 4      (B) -7      (C) -3      (D) 1      (E) NOTA

28. What is the equation of the line through the points (3, 1, 4) and (5, -2, -3)?

- (A)  $\frac{5-x}{3} = \frac{-y-2}{7} = \frac{z+3}{2}$       (B)  $-3x - 7y + 2z = -8$   
 (C)  $\frac{x-3}{2} = \frac{1-y}{3} = \frac{4-z}{7}$       (D)  $2x - 3y - 7z = -25$       (E) NOTA

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29. What is the inverse of  $\begin{bmatrix} 5 & 3 & -1 \\ -2 & 4 & 1 \\ 1 & 2 & a \end{bmatrix}$

(A)  $\frac{1}{26a+4} \begin{bmatrix} 4a-2 & -(3a+2) & 7 \\ 2a+2 & 5a+1 & -3 \\ -8 & -7 & 26 \end{bmatrix}$

(B)  $\frac{1}{26a+4} \begin{bmatrix} 3a+1 & a+3 & 7 \\ a-5 & 3a-2 & -3 \\ -8 & -7 & 26 \end{bmatrix}$

(C)  $\frac{1}{26a+4} \begin{bmatrix} a+1 & 2a-3 & 7 \\ 3a-5 & 2a-3 & -3 \\ -8 & -7 & 26 \end{bmatrix}$

(D)  $\frac{1}{26a+4} \begin{bmatrix} 3a-2 & a-3 & 7 \\ 4a+1 & 2a-3 & -3 \\ -8 & -7 & 26 \end{bmatrix}$

(E) NOTA

30. What is the area of the triangle formed by the points (1,3,5), (3,4,2), and (7,-2,-4)?

(A) 10

(B) 12

(C) 15

(D) 17

(E) NOTA