# ANSWER KEY FOR MATRIX TEST

1. A
2. C
3. D
4. B
5. B
6. A
7. B
8. B
9. D
10. D
11. C
12. C
13. E
14. A
15. C
16. E
17. C
18. A
19. A
20. E

# Solutions for Matrix Test

1. A

2. C  $2x_1 + 0 = -4$, $x_1 = -2$ or $-3x_1 + 12 = 18$, $x_1 = -2$ and $12 + 4x_2 = 32$, $x_2 = 5$

$$x_1 + x_2 = -2 + 5 = \boxed{3}$$

3. D

4. B

5. B  The minor of $-3$ is $\begin{vmatrix} 3 & -1 \\ -4 & 1 \end{vmatrix} = -1$ The Position of $-3$ ($2^{nd}$ row, $3^{rd}$ column) dictates sign change to $1$

6. A  Since 2 times the first row is equal to the $4^{th}$ row, the determinant is 0

7. B  The trace of a matrix is the sum of the elements in its main diagonal : $3+3+3=9$

8. B  A conic of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ will be degenerate if $\begin{vmatrix} 2A & B & D \\ B & 2C & E \\ D & E & 2F \end{vmatrix} = 0$

Substituting $\begin{vmatrix} 4 & 2 & -4 \\ 2 & 6 & 3 \\ -4 & 3 & 2F \end{vmatrix} = 0$  Expand by $3^{rd}$ row:
$-4(30) + 3(-20) + 40F = 0$
$F = 4.5$

9. D  A complex number $a+bi$ written in matrix form is $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$. Thus $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{12} = (1-i)^{12}$ In polar form,
$1 - i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right) = \sqrt{2} \operatorname{cis} \frac{7\pi}{4}$. Therefore,
$(1-i)^{12} = \left[\sqrt{2} \operatorname{cis} \frac{7\pi}{4}\right]^{12} = 64 \operatorname{cis} 21\pi = -64 + 0i = \begin{bmatrix} -64 & 0 \\ 0 & -64 \end{bmatrix}$

10. D  When doing the composition of two functions of the form $\frac{ax+b}{cx+d}$, the terms of the composite are the elements of the matrix product.
$f(g(x)) = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ 1 & 0 \end{bmatrix}$ which represents $\frac{7x-10}{x}$

11. C  The adjoint of a square matrix is the matrix of the cofactors of the transpose of the matrix. In this case, cofactors are found by multiplication and/or addition of natural numbers. Fractions cannot be produced. Most specific name is integers.

12. C $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ reflects points about the origin. Thus a figure in Quadrant I is reflected to Quadrant III

13. E The inverse of a rational function of the form $\frac{ax+b}{cx+d}$ is the adjoint of the coefficient matrix $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$.

Adjoint of $\begin{bmatrix}3 & -7\\ 4 & 2\end{bmatrix}$ is $\begin{bmatrix}2 & 7\\ -4 & 3\end{bmatrix}$ $\frac{2x+7}{-4x+3}$

14. A A diagonal matrix is a square matrix with 0's everywhere except on main diagonal

$$\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}\begin{bmatrix}d & 0 & 0\\ 0 & e & 0\\ 0 & 0 & f\end{bmatrix} = \begin{bmatrix}ad & 0 & 0\\ 0 & be & 0\\ 0 & 0 & cf\end{bmatrix}$$

15. C $\frac{1}{2}\begin{vmatrix}1 & 8 & 7 & 5 & 2 & 3 & 1\\ 1 & 3 & 7 & 5 & 9 & 4 & 1\end{vmatrix}$ $= \frac{1}{2}(3+56+35+45+8+3-8-21-35-10-27-4)$

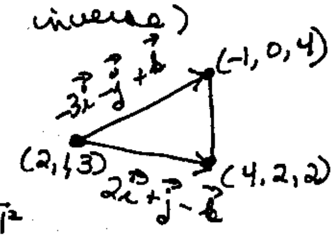
$= \frac{1}{2}(45) = 22.5$

16. E A matrix, not a value C $10(0)=0$ E. none of above have value of 100

B $10(0)=0$ D $100-1=99$

17. C Properties exhibited are closure and identity (matrix multiplication is not commutative; a matrix with determinant of 0 does not have an inverse)

18. A The area of the triangle is $\frac{1}{2}$ the magnitude of the cross product vector.




$\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -3 & -1 & 1\\ 2 & 1 & -1\end{vmatrix} = 0\vec{i}-\vec{j}+\vec{k}$ magnitude $= \sqrt{0^2+(-1)^2+1^2}$

$= \sqrt{2}$

divide by 2: $\sqrt{2}/2$

19. A A. Factor of 10 taken out of each row - number multiplied by determinant should be 1000

B. First row added to second row, no change in determinant

C. First column multiplied by 2 and added to third column, no change

D. First and second rows switched - must change sign of determinant

E. One factor of 10 taken out of one row.

20. E $\left[\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}-2 & 4\\ 3 & 5\end{bmatrix}\begin{bmatrix}-2 & 3\\ 4 & 5\end{bmatrix} + \begin{bmatrix}-2 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}-2 & 4\\ 3 & 5\end{bmatrix}\right] =$

$\left[\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}20 & 14\\ 14 & 34\end{bmatrix} + \begin{bmatrix}13 & 7\\ 7 & 41\end{bmatrix}\right] = \begin{bmatrix}35 & 21\\ 21 & 77\end{bmatrix}$

determinant is 2254