1999 Mu Alpha Theta National Convention
Logarithms & Exponents & Radicals
Alpha Division

1. If log tan x = m, then log cot x is equivalent to what?

Ans: B - m
Solution: Using the logarithmic properties, $\log \cot x = \log\left(\frac{1}{\tan x}\right) = \log 1 - \log \tan x = 0 - m = -m$

2. $(2^x)(4^x)$ = what ?

Ans: A 8^x
Solution: Using the power rule, $(2^x)(4^x) = 8^x$

3. Simplify: $\frac{3^{x-3}(3^{2x} - 3^{2x-4})}{27^{x-1}}$

Answer: A

Solution: $\frac{3^{x-3}(3^{2x} - 3^{2x-4})}{27^{x-1}} = \frac{3^{3x-3} - 3^{3x-7}}{3^{3x-3}} = 1 - 3^{-4} = 1 - \frac{1}{81} = \frac{80}{81}$

4. Find the sum of the roots of the equation: $8e^x + \frac{27}{e^x} = 35$

Answer: B

Solution: $8e^{2x} - 35e^x + 27 = 0$ $\quad e^x = \frac{35 \pm \sqrt{(-35)^2 - 4(8)(27)}}{16}$; $e^x = \frac{35 \pm 19}{16}$; $e^x = \frac{27}{8}$; $e^x = 1$

$x = \ln \frac{27}{8}$ $\quad x = \ln 1$; Sum $= \ln \frac{27}{8} + 0 = \ln \frac{27}{8}$

5. If $y = 2^n$, by what number is y multiplied when n is increased by 2?

Ans: B 4
Solution: $2^{n+2} = (2^n)(2^2) = 4(2^n) = 4y$

6. log sin 2x = ?

Answer: C
Solution: log sin 2x = log 2 sin x cos x = log 2 + log sin x + log cos x

7. Simplify: $[(4)^{\frac{-3}{2}}\sqrt{2^{12}} + (\frac{1}{8})^{\frac{-2}{3}}] \div (27)^{\frac{1}{3}}$

Ans: B
Solution: $[(4)^{\frac{-3}{2}}\sqrt{2^{12}} + (\frac{1}{8})^{\frac{-2}{3}}] \div (27)^{\frac{1}{3}} = [\frac{1}{8}(2^6) + 4] \div 3 = [2^3 + 4] \div 3 = 12 \div 3 = 4$

8. Solve: $\sqrt[4]{y} + \sqrt[4]{625y} = 2(1 + \sqrt[4]{y})$
Ans: C
Solution: $\sqrt[4]{y} + \sqrt[4]{625y} = 2(1 + \sqrt[4]{y})$ Simplifying you get

$\sqrt[4]{y} + 5\sqrt[4]{y} = 2 + 2\sqrt[4]{y}$ $\quad 4\sqrt[4]{y} = 2$ $\quad \sqrt[4]{y} = \frac{1}{2}$ $\quad y = \frac{1}{16}$

9. Simplify: $3 + \frac{1}{1 + \frac{1}{3 + \sqrt{15}}}$

Ans: C

Solution: $3 + \frac{1}{1 + \frac{1}{3 + \sqrt{15}}} =$

$$3 + \frac{1}{1 + \frac{3 - \sqrt{15}}{-6}} = 3 + \frac{1}{\frac{3 + \sqrt{15}}{6}} = 3 + \frac{6}{3 + \sqrt{15}} = 3 + \frac{18 - 6\sqrt{15}}{-6} = 3 - 3 + \sqrt{15} = \sqrt{15}$$

10. Simplify: $\sqrt{600} + \sqrt{864} - \sqrt{\frac{243}{3}}$

Ans: B

Solution: $\sqrt{600} + \sqrt{864} - \sqrt{\frac{243}{3}} = 10\sqrt{6} + 12\sqrt{6} - 9 = 22\sqrt{6} - 9$

11. The conjugate of $\sqrt[3]{a} - \sqrt[3]{b}$ is:

Ans: B

Solution: $(\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}) = a - b$

12. Simplify: $\sqrt{\sqrt[3]{\sqrt[4]{x^3}}}$

Ans: D

Solution: $\sqrt{\sqrt[3]{\sqrt[4]{x^3}}} = \sqrt{\sqrt[4]{\sqrt[3]{x^3}}} = \sqrt{\sqrt[4]{x}} = \sqrt[8]{x}$

13. Find the value of $\frac{x}{y}$ if $\frac{3}{\sqrt{y}} - \frac{1}{\sqrt{x}} = \frac{2}{\sqrt{x} + \sqrt{y}}$

Answer: A

Solution: $\frac{3}{\sqrt{y}} - \frac{1}{\sqrt{x}} = \frac{2}{\sqrt{x} + \sqrt{y}}$; $\frac{3\sqrt{x} - \sqrt{y}}{\sqrt{xy}} = \frac{2}{\sqrt{x} + \sqrt{y}}$; $3x - \sqrt{xy} + 3\sqrt{xy} - y = 2\sqrt{xy}$

$3x - y = 0 \quad x = \frac{y}{3}$ $\qquad \frac{x}{y} = \frac{\frac{y}{3}}{y} = \frac{1}{3}$

14. The expression $\sqrt{8+\sqrt{8+\sqrt{8+\sqrt{8+\ldots}}}}$ where the dots indicate an infinite repetition of the pattern can be expressed in the form $\frac{a+\sqrt{b}}{c}$. Find a + b + c.

Ans: C
Solution: Let x = $\sqrt{8+\sqrt{8+\sqrt{8+\sqrt{8+\ldots}}}}$, then $x = \sqrt{8+x}$ $x^2 = 8 + x$ $x^2 - x - 8 = 0$ Using the quadratic formula: $x = \frac{1 \pm \sqrt{33}}{2}$. Therefore, a + b + c = 1 + 33 + 2 = 36

15. Consider the equation $\log(x + \pi) = \log x + \log \pi$, where x is a positive real number. This equation has:

Ans: B
Solution: Since $\log(x + \pi) = \log(\pi x)$, then $x + \pi = \pi x$; $\pi x - x = \pi$; $x = \frac{\pi}{\pi - 1}$

16. Let $T = \frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$ then

Answer: D
Solution: By rationalizing the denominator of each fraction, you get

$T = (3+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+2) = 3 + 2 = 5$

17. Which positive numbers x satisfy the equation $(\log_3 x)(\log_x 5) = \log_3 5$?

Answer: C
Solution: Converting all these logs to some fixed but arbitrary base you get

$(\log_3 x)(\log_x 5) = \frac{\log_d x}{\log_d 3} \cdot \frac{\log_d 5}{\log_d x} = \frac{\log_d 5}{\log_d 3} = \log_3 5$ for all $x \neq 1$

18. The product of $\sqrt[3]{4}$ and $\sqrt[4]{8}$ equals

Answer: D
Solution: $\sqrt[3]{4}\,\sqrt[4]{8} = (2^2)^{\frac{1}{3}} \cdot (2^3)^{\frac{1}{4}} = 2^{\frac{2}{3}} \cdot 2^{\frac{3}{4}} = 2^{\frac{17}{12}} = 2 \cdot 2^{\frac{5}{12}} = 2(\sqrt[12]{32})$

19. Let a, b, and x be positive real numbers distinct from one. Then
$4(\log_a x)^2 + 3(\log_b x)^2 = 8(\log_a x)(\log_b x)$

Answer: E
Solution: the given equation may be written in the form
$4(\log_a x)^2 - 8(\log_a x)(\log_b x) + 3(\log_b x)^2 = 0$ Factoring: $(2\log_a x - \log_b x)(2\log_a x - 3\log_b x) = 0$
$\log_a x^2 = \log_b x$ or $\log_a x^2 = \log_b x^3$

Let $r = \log_a x^2$. Then $a^r = x^2$ and $b^r = x$ or $a^r = x^2$ and $b^r = x^3$
$a^r = b^{2r}$ or $a^{3r} = b^{2r}$

Since $x \neq 1$ we have $r \neq 0$ and $a = b^2$ or $a^3 = b^2$

20. If $\sqrt{x+2} = 2$ then $(x+2)^2$ equals

Answer: D
Solution: $x + 2 = 4$; $(x + 2)^2 = 16$

21. For all positive numbers x distinct from 1, $\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x}$

Answer: A
Solution: Since $\log_b a = \frac{1}{\log_a b}$ then

$$\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} = \log_x 3 + \log_x 4 + \log_x 5 = \log_x 60 = \frac{1}{\log_{60} x}$$

22. The fraction $\frac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}}$ is equal to

Answer: C
Solution: If $x = \frac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}}$, then

$$x^2 = \frac{4(2+2\sqrt{12}+6)}{9(2+\sqrt{3})} = \frac{4(8+4\sqrt{3})}{9(2+\sqrt{3})} = \frac{16(2+\sqrt{3})}{9(2+\sqrt{3})} = \frac{16}{9} \qquad x = \frac{4}{3}$$

23. Which of the following (where all exponents are integers) is NOT true?

Answer: D
Solution: The only one not true is: $(x^{-1} + 2y)^{-1} = (\frac{1}{x} + 2y)^{-1} = (\frac{1+2y}{x})^{-1} = \frac{x}{1+2y}$

24. Which of the following is not ALWAYS true?

Answer: A
Solution: $\sqrt{x^2y^6} = |xy^3|$

25. Determine the value(s) of x in the expression: $\log_x x^{x^2} + \log_x x^{-5x} = \log_x \frac{1}{x^6}$

Answer: B

Solution: $\log_x x^{x^2} + \log_x x^{-5x} = \log_x \frac{1}{x^6}$ $\quad X^2 - 5x + 6 = 0$ $\quad (x-2)(x-3)=0$ $\quad X = 2, x = 3$

26. If $3^{x+2} = 2^x$, log 3 = a and log 2 = b, solve for x in terms of a and b.

Answer: B
Solution: $3^{x+2} = 2^x$ $\quad (x + 2) \log 3 = x \log 2$ $\quad (x + 2)(a) = x (b)$ $\quad x(a - b) = - 2a$ $\quad x = \frac{-2a}{a-b}$

27. Simplify: $\sqrt{5 + 2\sqrt{6}}$

Answer: D
Solution: $\sqrt{a} + \sqrt{b} = \sqrt{5 + 2\sqrt{6}}$; $a + 2\sqrt{ab} + b = 5 + 2\sqrt{6}$ a + b = 5; ab = 6 a=3, b=2

28. If log 36 = a and log 125 = b, express $\log(\frac{1}{12})$ in terms of a and b.

Answer: D
Solution: $\log 6 = \frac{a}{2}$; $\log 5 = \frac{b}{3}$; $\log 2 = \log 10 - \log 5 = \frac{3-b}{3}$

$\log\frac{1}{12} = -\log 12$; $-[\log 6 + \log 2] = -[\frac{a}{2} + \frac{3-b}{3}] = \frac{2b - 3a - 6}{6}$

29. Find the fourth term of the expansion of $(x\log_8 2 - y\log_3 9)^5$

Answer: C
Solution: $\log_8 2 = \frac{1}{3}$; $\log_3 9 = 2$; $(\frac{x}{3} - 2y)^5$ $\;_5C_2(\frac{x}{3})^2(-2y)^3 = 10(\frac{x^2}{9})(-8y^3) = \frac{-80}{9}x^2y^3$

30. Simplify: $\sqrt[3]{\frac{\sqrt{112} - \sqrt{9072}}{\sqrt{112}}} - \sqrt[4]{\frac{\sqrt{1008} - \sqrt{448}}{\sqrt{112}}}$

Answer: B
Solution: $\sqrt[3]{\frac{\sqrt{112} - \sqrt{9072}}{\sqrt{112}}} - \sqrt[4]{\frac{\sqrt{1008} - \sqrt{448}}{\sqrt{112}}} =$

$\sqrt[3]{1 - \sqrt{81}} - \sqrt[4]{\sqrt{9} - \sqrt{4}} = \sqrt[3]{-8} - \sqrt[4]{1} = -2 - 1 = -3$

Tiebreaker Questions:

T1. Find the product of the positive numbers a, b and c when:

$$(a + b + c)^{-1}(a^{-1} + b^{-1} + c^{-1})(ab + bc + ac)^{-1}((ab)^{-1} + (bc)^{-1} + (ac)^{-1}) = \frac{9}{16}$$

Solution: $(a + b + c)^{-1}(\frac{bc + ac + ab}{abc})(ab + bc + ac)^{-1}(\frac{c + a + b}{abc}) = \frac{9}{16}$

$$(\frac{1}{abc})^2 = \frac{9}{16} \quad \therefore \quad abc = \frac{4}{3}$$

T2. In interval notation, state the domain of $f(x) = \log_3|x|$?

Answer: $(-\infty, 0) \cup (0, \infty)$
Solution: $\log_3|x|$ Is defined for all real numbers except 0. In interval notation this is:
$(-\infty, 0) \cup (0, \infty)$

T3. For what value(s) of x will $\sqrt[3]{x^3 + 6x^2 - 4} - x - 2 = 0$ be true?

Answer: x = - 1

Solution: $\sqrt[3]{x^3 + 6x^2 - 4} = x + 2$ $x^3 + 6x^2 - 4 = (x + 2)^3$; $x^3 + 6x^2 - 4 = x^3 + 6x^2 + 12x + 8$;
$-12 = 12x$ $x = -1$