1992 National Mu Alpha Theta Convention

Logarithms & Exponents Topic Test Answers:

1. A	16. E
2. B	17. B
3. B	18. C
4. B	19. A
5. C	20. D
6. E	21. A
7. D	22. A
8. A	23. E
9. C	24. C
10. D	25. D
11. E	26. A
12. A	27. C
13. A	28. B
14. D	29. C
15. D	30. A

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(1) $\log_2 8 = x \quad 2^x = 8 \quad 2^x = 2^3 \quad x = 3$ A

(2) $a = \log_8 225 = \frac{\log 225}{\log 8} = \frac{2\log 15}{3\log 2} = \frac{2}{3}\log_2 15 =$

$\frac{2}{3}b$ B

(3) $(-i)^{3895} = -(i)^{3895} = -(i)^3 = -(-i) = i$ B

(4) $f(a) = \log_a a = \frac{\log a}{\log a} = 1$

$f(1) = \log_a 1 = 0 \quad (a^0 = 1)$

$f(0) = \log_a 0$ is not defined B

(5) $4^x = 17$ in log form is $\log_4 17 = x$

$\frac{\log 17}{\log 4} = \frac{\log 17}{2\log 2} = \frac{1}{2}(\log 17)/(\log 2)$ C

(6) $\log(x^3 - y^3) - \log(x - y) = \log\left(\frac{x^3 - y^3}{x - y}\right) =$

$\log \frac{(x-y)(x^2 + xy + y^2)}{(x-y)} = \log(x^2 + xy + y^2)$ E

(7) a) $-(8/27)^{-2/3} = -4/3$ $\frac{1}{3}$ away from -1

b) $-(8/27)^{2/3} = -3/4$ $\frac{1}{4}$ away

c) $-\left(\frac{16}{25}\right)^{-1/2} = -5/4$ $\frac{1}{4}$ away

d) $-\left(\frac{16}{25}\right)^{+1/2} = -4/5$ $\frac{1}{5}$ away D

(8) $(256)^{0.15} \cdot (256)^{0.1} = (256)^{0.25} = (2^8)^{1/4} = 2^2 = 4$ A

(9) $\left[(a^{-2} - b^{-2}) \div (a+b)\right]^{-1} = \left[\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \cdot \frac{1}{(a+b)}\right]^{-1}$

$\left[\frac{(b^2 - a^2)}{a^2b^2} \cdot \frac{1}{(a+b)}\right]^{-1} = \left[\frac{-(a-b)(a+b)}{a^2b^2} \cdot \frac{1}{(a+b)}\right]^{-1} =$

$-\frac{a^2b^2}{a-b}$ C

(10) $36e^x + 6e^{-x} = 35$ Mult. by e^x

$36e^{2x} + 6 = 35e^x$

$36e^{2x} - 35e^x + 6 = 0$

$(9e^x - 2)(4e^x - 3) = 0$

$e^x = 2/9 \quad e^x = 3/4 \quad \{\ln 2/9, \ln 3/4\}$

sum of roots: $\ln 2/9 + \ln 3/4 = \ln\left(\frac{2}{9}\right)\left(\frac{3}{4}\right) =$

$\ln \frac{1}{6} = \ln 6^{-1} = -\ln 6$ D

(11) $2^a = 256 \quad 2^a = 2^8 \quad a = 8$

$3^b = 27 \quad 3^b = 3^3 \quad b = 3$

$(2bx^3 - ay)^9 = (6x^3 - 8y)^9 =$ seven term polynomial with the sum of the coefficients equal to $(6-8)^9 = (-2)^9 = -512$. E

(12) $-2^{2^3} = -(2)^8 = -256$ A

(13) $\frac{(2n^{1/3} - 4n^{-2/3})}{2n^{-2/3}} \cdot \frac{n^{2/3}}{n^{2/3}} = \frac{2n-4}{2} = n-2$ A

(14) D $\quad 2^{-(x+3)} + 1 = y$

$\left(\frac{1}{2}\right)^{x+3} + 1 = y$

shifted 3 to left and up 1.

(15) $(16)^{20}(125)^{27} = 2^{80} \cdot 5^{81} = (2 \cdot 5)^{80} \cdot 5 =$

$10^{80} \cdot 5 \qquad 10^1$ has 2 digits

5×10^{80} has 81 digits D

(16) $\left[\frac{2x^2y^{-1}}{y^{-3}}\right]^4 = \left[2x^2y^2\right]^4 = 16x^8y^8$ E

(17) $\log_A B = \log_B A \qquad \frac{\log B}{\log A} = \frac{\log A}{\log B}$

$[\log B]^2 = [\log A]^2$ a − number squared = a + number squared

$[\log B]^2 = \left[\log \frac{1}{B}\right]^2 \quad A = \frac{1}{B} \quad \frac{1}{B} \cdot B = 1$

B

(18) $\left(\frac{1}{49}\right)^{2x-3} = 343^{3x-2}$

$(7^{-2})^{2x-3} = (7^3)^{3x-2} \qquad -4x + 6 = 9x - 6$

$12 = 13x$

$\frac{12}{13} = x$ C

(19) $3^6 = 729 \quad [\log_3 769] = 6$

$6^3 = 216 \quad 6^4 = 1296 \quad [\log_6 1024] = 3$

$5^5 = 3125 \quad 5^6 = 15635 \quad [\log_5 15124] = 5$

$[\log 1.9647 \times 10^{-5}] = -5$

$7^3 = 343 \quad 7^4 = 2301 \quad [\log_7 1299] = 3$

$6 + 3 + 5 - 5 + 3 = 12$ A

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(20) $\frac{\log\frac{1}{25}}{\log 8}\cdot\frac{\log 4096}{\log 9}\cdot\frac{\log 36}{\log 625}\cdot\frac{\log 2}{\log 6} =$

$\frac{-2\log 5}{3\log 2}\cdot\frac{12\log 2}{2\log 3}\cdot\frac{2\log 6}{4\log 5}\cdot\frac{\log 2}{\log 6} =$

$\frac{-2\log 2}{\log 3} = \frac{\log 2^{-2}}{\log 3} = \frac{\log 1/4}{\log 3} = \log_3 0.25$ D

(21) $3^{\log x} = 3x$ $\qquad \log 3^{\log x} = \log(3x)$

$\log x \cdot \log 3 = \log 3 + \log x$

$\log x \cdot \log 3 - \log x = \log 3$

$\log x\,(\log 3 - 1) = \log 3$

$\log x = \frac{\log 3}{\log 3 - \log 10}$ $\quad$ mult by $\frac{1}{\log 3}$

$\frac{\log x}{\log 3} = \frac{1}{\log 3 - \log 10}$

$\log_3 x = \frac{\log 10}{\log 3/10}$ $\quad \left[\frac{\log 10}{\log 3/10} = \frac{\log_3 10}{\log_3 3/10}\right]$

$\log_3 x = \frac{\log_3 10}{\log_3 0.3}$

$3^{(\log_3 10)/\log_3 0.3} = x$ $\qquad$ A

(22) $\frac{\log a}{\log 4} + \log_{64} b = c$

$\frac{3\log a}{3\log 4} + \log_{64} b = c$

$\log_{64} a^3 + \log_{64} b = c$

$\log_{64} a^3 b = c$ $\qquad a^3 b = 32$

$\log_{64} 32 = c$ $\qquad 64^c = 32$ $\qquad 2^{6c} = 2^5$ $\qquad c = \frac{5}{6}$ A

(23) $x^{4/3} - 6x^{2/3} + 8 = 0$

$(x^{2/3} - 4)(x^{2/3} - 2) = 0$

$x^{2/3} = 4$ $\qquad x^{2/3} = 2$

$x = \pm(4)^{3/2}$ $\qquad x = \pm(2)^{3/2}$

$\{-8, +8, \sqrt{2}, -2\sqrt{2}\}$

sum of roots $= 0$ E

(24) (a) $2^{x+2} - 2^x = 2^2\cdot 2^x - 2^x = 2^x(4-1) = 3\cdot 2^x$

(b) $2^{x+5} \div 2^x = 2^{x+5-x} = 2^5 = 32$

(c) $2^{14} = 16384$

(d) $2^{x+3} - 2^x = 2^3\cdot 2^x - 2^x = 2^x(2^3-1) = 7\cdot 2^x$

C is true

(25) $x^{(x+3)^2} = x^{64}$ $\qquad -1, 0, 1$ all give true statements

$(x+3)^2 = 64$

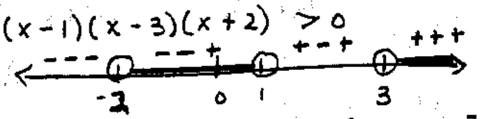
$x + 3 = \pm 8$ $\qquad x = -3 \pm 8$ $\qquad 5, -11$

$|-1| + |0| + |1| + |5| + |-11| = 18$ D

(26) $x^3 - 2x^2 - 5x + 6 > 0$

$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$ $\qquad (x^2 - x - 6)$
$\qquad (x-3)(x+2)$

$(x-1)(x-3)(x+2) > 0$



$\{x : -2 < x < 1\} \cup \{x : x > 3\}$ A

(27) $3 + 1 + 1 + \frac{1}{4} + \frac{1}{3} + \frac{1}{16} + \frac{1}{9} + \ldots$

two infinite series

$3 + 1 + \frac{1}{3} + \frac{1}{9} + \ldots$ $\qquad \frac{3}{1 - 1/3} = 3(3/2) = 9/2$

$1 + 1/4 + \frac{1}{16} + \ldots$ $\qquad \frac{1}{1 - 1/4} = 1\cdot 4/3 = 4/3$

$\frac{9}{2} + \frac{4}{3} = \frac{27+8}{6} = \frac{35}{6}$ C

(28) $\log_9 a = \log_{15} b = \log_{25}(a+2b) = x$ $\qquad \frac{b}{a} = \left(\frac{15}{9}\right)^x = \left(\frac{5}{3}\right)^x$

$9^x = a$ $\quad 15^x = b$ $\quad 25^x = a + 2b$

$9^x + 2\cdot 15^x = 25^x$ $\qquad$ Mult. by $\frac{1}{9^x}$

$1 + 2\left(\frac{15}{9}\right)^x = \left(\frac{25}{9}\right)^x$ $\qquad 1 + 2\left(\frac{5}{3}\right)^x = \left(\left(\frac{5}{3}\right)^x\right)^2$

$\left[\left(\frac{5}{3}\right)^x\right]^2 - 2\left[\frac{5}{3}\right]^x - 1 = 0$ $\qquad \left(\frac{5}{3}\right)^x = \frac{2 \pm \sqrt{4 - 4(1)}}{2}$

$\left(\frac{5}{3}\right)^x = 1 \pm \sqrt{2}$ $\qquad \frac{b}{a} > 0$ so $\frac{b}{a} = 1 + \sqrt{2}$ B

(29) $A = Pe^{rt}$ $\qquad A = 600{,}000\, e^{10(.1)} = 600{,}000(e) \approx$

$600{,}000 \cdot 2.7 \approx 1.6$ million $\qquad$ C

(30) $e^{\ln 6 - \ln 2} = e^{\ln 6/2} = e^{\log_e 3} = 3$ $\qquad \begin{vmatrix} 3 & 9 \\ 18 & 21 \end{vmatrix} =$

$e^{\ln 9 + \ln 1} = e^{\ln 9} = 9$

$e^{2\ln 3 + \frac{1}{2}\ln 4} = e^{\ln 9\cdot 2} = 18$ $\qquad 3(21) - 9(18) =$

$e^{\ln 7 + \ln 3} = e^{\ln 21} = 21$ $\qquad 63 - 162 = -99$

A