Logs and Exponents Topic Test Answers

1. E
2. B
3. C
4. C
5. A
6. E
7. B
8. A
9. B
10. A
11. C
12. E
13. E
14. E
15. B
16. A
17. A
18. C
19. A
20. E
21. D
22. B
23. E
24. C
25. A

$\boxed{6}$

Log and Exponential Topic Test Solutions

① $x-1 = \log_3 81$ ans. E

$3^{x-1} = 3^4$

$x = 5$

② $\log_2(x+6) + \log_2(x-6) = 6$ ans B

$x^2 - 36 = 2^6$

$x^2 = 100$

$x = 10$

③ $\dfrac{2^{3x-2} \cdot 4^{x-2}}{32^{x-1}} =$ ans C

$\dfrac{2^{3x-2} \cdot 2^{2x-4}}{2^{5x-5}} = \dfrac{1}{2}$

④ $a = \log_4 7$ ans C

$4^a = 7$

$2^{2a} = 7$

$2^a = \sqrt{7}$

⑤ $(1+i)^{20} = \left[(1+i)^2\right]^{10}$ ans A

$= (2i)^{10}$

$= -1024$

⑥ $3^{x+4y} = \left(\frac{1}{9}\right)^{y-2x}$ ans E

$3^{x+4y} = 3^{-2y+4x}$

$x+4y = -2y+4x$

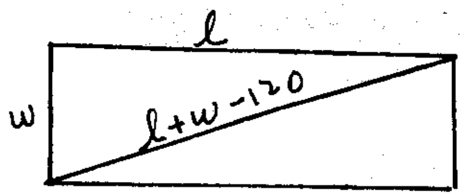
$\frac{x}{y} = \frac{2}{1}$

⑦ $10^a (20)^b = 4000$ ans B

$10^a\, 20^b = 10^1 \cdot 20^2$

$a+b = 1+2 = 3$

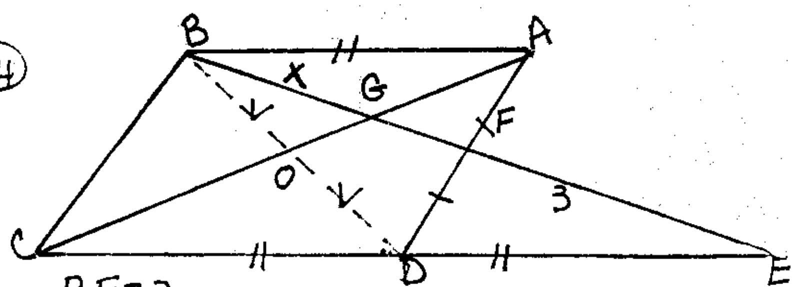
(23)



ans E

$$lw = 42000$$
$$l^2 + w^2 = l^2 + 2lw + w^2 - 240l - 240w + 14400$$
$$l + w = \frac{98400}{240}$$
$$l + w = 410$$
$$l + w - 120 = 410 - 120$$
$$= 290$$

(24)

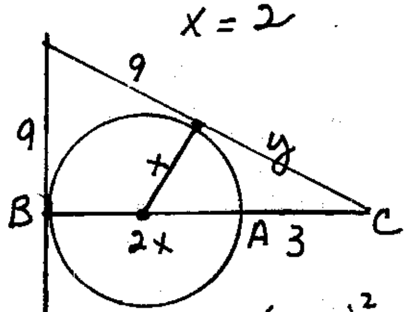


ans. A

BF = 3
FE = 3
BE = 6
BF and AO medians of $\triangle BDA$
Med. intersect in a ratio of 2:1.
Let $BG = x$ $\therefore GE = 2x$

$$x + 2x = 6$$
$$3x = 6$$
$$x = 2$$

(25)



$$(2x+3)^2 + 9^2 = (9+y)^2$$
$$x^2 + y^2 = (x+3)^2$$
$$4x^2 + 12x + 9 + \cancel{81} = \cancel{81} + 18y + y^2$$
$$\cancel{x^2} + y^2 = \cancel{x^2} + 6x + 9$$
$$4x^2 + 12x + \cancel{9} = 18\sqrt{6x+9} + 6x + \cancel{9}$$
$$2x^2 + 3x = 9\sqrt{6x+9}$$

$$4x^4 + 12x^3 + 9x^2 = 486x + 729$$
$$4x^4 + 12x^3 + 9x^2 - 486x - 729 = 0$$

$\frac{9}{2}$	4	12	9	-486	-729
		18	135	648	729
	4	30	144	162	0

$$x = \frac{9}{2}$$
$$2x = 9$$

ans. C

(8) $b^a = a^b$ ans A

$\log_b b^a = \log_b a^b$

$a = b \log_b a$

$\log_b a = \dfrac{a}{b}$

(9) $7r + 6 = r^3$

$r^3 - 7r + 6 = 0$ 2 pos and 1 neg root ans B

3 roots

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x \neq -3$

$(x+3)(x-1) = 0$ $x \neq 1$

(10) $\log_b a = x$

$b^x = a$

$\log_a b^x = \log_a a$ ans A

$x \log_a b = 1$

$\dfrac{\log_a b}{\log_b a} = \dfrac{\frac{1}{x}}{x}$

$\dfrac{\log_a b}{\log_b a} = \dfrac{1}{x^2}$

ans C

(11) $y = 2^{x-1} - 2$ $2 + (-\frac{3}{2}) = \frac{1}{2}$

y int $= -\frac{3}{2}$

x int $= 2$

(12) $x^{2y} + x^{-2y} - 2 = 0$ Ans E

$x^{4y} - 2x^{2y} + 1 = 0$

$(x^{2y} - 1)^2 = 0$

$x^{2y} = 1$

If $y = 0$ $x \in R$

$\therefore$ can't be determined

(13) $\log_5 2 = a$

$\log_5 3 = b$ ans E

$\log_{25} 96 = \log_5 (2^5 \cdot 3^1)^{\frac{1}{2}}$

$= \frac{1}{2}[5 \log_5 2 + \log_5 3]$

$= \frac{5a+b}{2}$

(14) $\left(\frac{1}{a}\right)^{\frac{1}{b}} = x$

$(a^{-1})^{b^{-1}} = x$ ans E

$\frac{1}{a} = x^b$

$a = \frac{1}{x^b}$

$a^b = \left(\frac{1}{x^b}\right)^b$

$a^b = \left(\frac{1}{x}\right)^{b^2}$

(15) $3^{20} > 32^x$ ans B

$3^{20} > 2^{5x}$

$3^4 > 2^x$

$x \leq 6$

(16) $\log_9 27 (243)^{\frac{1}{2}} = x$ ans A

$3^{2x} = 3^3 \cdot 3^{\frac{5}{2}}$

$2x = \frac{11}{2}$

$x = \frac{11}{4}$

(17) $3^{2x-1} - 6\cdot 3^{x} + 27 = 0$

$3^{2x} - 18\cdot 3^{x} + 81 = 0$

$(3^{x} - 9)^{2} = 0$

$3^{x} = 9$

$x = 2$

ans A

(18) $27^{x} - 2(3)^{2x+2} + 4(3)^{x+3} - 216 = 0$

$3^{3x} - 18\cdot 3^{2x} + 108\cdot 3^{x} - 2^{3}\cdot 3^{3} = 0$

$(3^{x} - 6)^{3} = 0$

$3^{x} = 6$

$x = \log_3 6$

ans C

(19) $(1-2x)^{\frac{1}{3}}$

4^{th} term $\binom{\frac{1}{3}}{3}(1)^{-\frac{8}{3}}(-2x)^{3}$

$\dfrac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)(1)^{-\frac{8}{3}}(-2x)^{3}}{1\cdot 2\cdot 3}$

$\dfrac{\frac{10}{27}}{6}\cdot(-8)$

$\dfrac{\cancel{10}^{\,5}\cdot 1}{27\cdot \cancel{6}_{\,3}}\cdot(-8) = \dfrac{-40}{81}$

ans A

(20) Let $a = \log_{4n} 40\sqrt{3}$

$\therefore a = \log_{3n} 45$

$4^{a} n^{a} = 40\sqrt{3}$

$3^{a} n^{a} = 45$

$\left(\frac{4}{3}\right)^{a} = \frac{8\sqrt{3}}{9}$

$\dfrac{2^{2a}}{3^{a}} = \dfrac{2^{3}}{3^{\frac{3}{2}}}$

$\therefore a = \frac{3}{2}$

$3^{\frac{3}{2}} n^{\frac{3}{2}} = 45$

$n^{3} = 75$

ans E

(21) $x = 5^{44}$

$\log x = 44 \log 5$ ans. D

$\log x = 44(\log 10 - \log 2)$

$\log x = 44(1 - .201)$

$\log x = 30.756$

no. of digits is 31

(22) $2^a + 2^b = 3^c$

2 to any power except 0 is even ans B

3 to any power is odd.

a or b must equal 0.

(23) $\ln x^4 = (\ln x)^3$

$(\ln x)^3 - 4\ln x = 0$ ans E

$\ln x(\ln x - 2)(\ln x + 2) = 0$

$\ln x = 0$ or $\ln x = \pm 2$

$x = e^0$ $\quad x = e^{\pm 2}$

$x = 1$ $\quad x = e^2, \frac{1}{e^2}$

(24) $11 > a > b > c > 0$ ans. C

$\log a + \log b > \log c > 0 \rightarrow ab > c$

$\log b + \log c > \log a \rightarrow bc > a$

$\log a + \log c > \log b \rightarrow ac > b$

Simply list them

with $c = 2$ 19 Δ

$c = 3$ 21 Δ

$c = 4$ 15

$c = 5$ 10

$c = 6$ 6

$c = 7$ 3

$c = 8$ 1

75

(25) $\sum_{n=0}^{\infty} \frac{\cos nx}{2^n}$ where $\cos x = \frac{1}{5}$

$\cos nx$ = real part of e^{inx} B

$\therefore \sum_{n=0}^{\infty} \left(\frac{e^{ix}}{2}\right)^n$ This is geometric.

$$\therefore S = \frac{1}{1 - \frac{e^{ix}}{2}} = \frac{2}{2 - e^{ix}} = \frac{2}{2 - \cos x - i \sin x}$$

$$= \frac{2(2 - \cos x + i \sin x)}{(2 - \cos x - i \sin x)(2 - \cos x + i \sin x)}$$

$$= \frac{2(2 - \cos x + i \sin x)}{4 - 4\cos x + \cos^2 x + \sin^2 x}$$

$$= \frac{2(2 - \cos x + i \sin x)}{5 - 4 \cos x}$$

Take real part and evaluate

$$= \frac{2(2 - \frac{1}{5})}{5 - 4(\frac{1}{5})}$$

$$= \frac{18}{21}$$

$$= \frac{6}{7}$$