Gaither-Leto Invitational Calculus Bowl 2/22/97

1. $h'(3) = 5'(g(3))\cdot g'(3) + 4(5(3))^{-2}\cdot 5'(3) + \tfrac{1}{2}(g(3))^{-1/2}\cdot g'(3) + 5(3)g'(3) + g(3)\cdot 5'(3)$
$= (-1)(4) - 4(1/4)(-1) + \tfrac{1}{2}\left(\tfrac{1}{\sqrt{3}}\right)(4) + 2(4) + 3(-1) = -4+1+\tfrac{2\sqrt{3}}{3}+8-3 = \boxed{\tfrac{2+2\sqrt{3}}{3}}$ or $\boxed{\tfrac{6+2\sqrt{3}}{3}}$

2. $c\int_0^2 \frac{x}{x^2+4}\,dx = 1$ $\quad u = x^2+4$, $du = 2x\,dx$ $\quad \frac{c}{2}\int_4^8 \frac{1}{u}\,du = \frac{c}{2}(\ln 8 - \ln 4)$ $\quad \frac{c\cdot \ln 2}{2} = 1$ $\quad \boxed{c = \frac{2}{\ln 2}}$

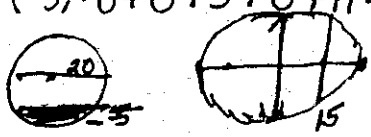
3. Use calculator $(-2, .17)$, $(.54, 3.55)$ $(1.84, -5.28)$

4. $\ln y = (2x-2)\ln(x^2-1)$
$\frac{y'}{y} = (2x-2)\cdot\frac{2x}{x^2-1} + \ln(x^2-1)\cdot 2$
$y' = \left[\frac{(2x-2)(2x)}{x^2-1} + 2\ln(x^2-1)\right]\cdot y$ $\left[\frac{8}{3}+2\ln 3\right]3^2 = 24+18\ln 3$ $\quad a+b = \boxed{42}$

5. $A = 2x(x^6+1)$ $\quad B = -1(x^3+1)^{-2}\cdot 3x^2$ $\quad C$ $\quad u = t^3+1$, $du = 3t^2\,dt$ $\quad \frac{1}{3}\int u^2\,du = \left[\frac{u^3}{9} = \frac{(x^3+1)^3}{9}\right]_0^{x^3+1}$
$D = \frac{6x^5}{x^6+1}$ $\quad 2x(x^6+1)\cdot\frac{-3x^2}{(x^3+1)^2}\cdot\frac{(x^3+1)^3}{9}\cdot\frac{6x^5}{x^6+1} = \boxed{-4x^8(x^3+1) \text{ or } -4x^{11}-4x^8}$

6. $y' = 4x^3 - 24x = 4x(x^2-6)$ $\quad y'' = 12x^2 - 24 = 12(x^2-2)$ $\quad x = \sqrt{2}$ $\quad x = -\sqrt{2}$
$m_{\tan} = 4(\sqrt{2})(-4) = -16\sqrt{2}$
$m_{\tan} = 4(-\sqrt{2})(-4) = 16\sqrt{2}$
$\tan\theta = \frac{-16\sqrt{2} - (+16\sqrt{2})}{1+(16\sqrt{2})(-16\sqrt{2})} = \frac{-32\sqrt{2}}{-511} = \boxed{\frac{32\sqrt{2}}{511}}$

7. $(-3)+0+0+5+0+11 = \boxed{13}$

8. 

$x^2+y^2 = 400$
$y^2 = 400 - x^2$
$\pi\int_{15}^{20}(400-x^2)\,dx = \boxed{1439.9}$

9. $\frac{3}{12}\left[1 + 2(1.12) + 2(1.41) + 2(1.8) + 2(2.24) + 2(2.69) + 3.16\right] = \boxed{5.67}$

10. $\int_0^a \sin x\, 2^{\cos x}\,dx$ $\quad u = \cos x$, $du = -\sin x\,dx$ $\quad -\int_1^{\cos a} 2^u\,du = -\left|_1^{\cos a} \frac{2^u}{\ln 2} = -\left[\frac{2^{\cos a}-2}{\ln 2}\right] = \frac{3}{2\ln 2}\right.$
$2^{\cos a} - 2 = -\frac{3}{2}$ $\quad 2^{\cos a} = \frac{1}{2}$ $\quad \cos a = -1$ $\quad \boxed{a = \pi}$

11. $v(t) = -\frac{t^3}{3} + C$ $\quad v(b) = 0$ $\quad C = \frac{b^3}{3}$ $\quad x(0) = 0$ $\quad x(t) = -\frac{t^4}{12} + \frac{b^3}{3}t$
$v(2b) = \frac{-8b^3}{3} + \frac{b^3}{3} = \frac{-7b^3}{3}$ $\quad x(b) = \frac{-b^4}{12} + \frac{b^4}{3} = \frac{3b^4}{12}$ $\quad \frac{-7b^3}{3}\cdot\frac{4}{b^4} = \boxed{\frac{-28}{3b}}$

12. 

$A = \frac{x}{2\pi}\cdot\pi r^2 = \frac{xr^2}{2}$ $\quad P = 2r + \frac{x}{2\pi}\cdot 2\pi r = 2r + xr = 2r + 2Ar^{-1}$
$2A = xr^2$ $\quad x = \frac{2A}{r^2}$ $\quad P'(r) = 2 - 2Ar^{-2}$ $\quad 2r^2 = 2A$ $\quad \boxed{r = \sqrt{A}}$
$2 = 2Ar^{-2}$

13. $2\pi\int_1^4 (5-x)\sqrt{x}\,dx = 2\pi\int_1^4 (5x^{1/2} - x^{3/2})\,dx = 2\pi\left|_1^4 \frac{10}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right.$
$2\pi\left[\left(\frac{80}{3} - \frac{64}{5}\right) - \left(\frac{10}{3} - \frac{2}{5}\right)\right] = 2\pi\left[\frac{70}{3} - \frac{62}{5}\right] = 2\pi\left[\frac{350-186}{15}\right] = \boxed{\frac{328\pi}{15}}$

14. $dy = \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2\cdot b}\cdot 2 = \frac{1}{b}$ $\quad b + \frac{1}{b} = \frac{b^2+1}{b}$

15. $2x + 2yy' = 0$ $\quad x^2 + y^2 = 12$ $\quad -x^2 - y^2 = -12$
$y' = -\frac{x}{y}$ $\quad y'' = \frac{y(-1) - (-x)y'}{y^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} = \frac{-y^2 - x^2}{y^3} = \boxed{\frac{-12}{y^3}}$