'94 Sauver - Zelo Invitational Calculus Individual

B 1. $\lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0$

A 2. $1 = x^2+2x-2 \quad x^2+2x-2=0 \quad \frac{-2\pm\sqrt{4-4(1)(-2)}}{2} = \frac{-2\pm\sqrt{12}}{2} = \frac{-2\pm 2\sqrt{3}}{2} = -1\pm\sqrt{3}$

A 3. $A(x) = \frac{1}{2}ab\sin x \quad \frac{dA}{dt} = 18\cos x \frac{dx}{dt} = 18\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\pi}{90}\right) = \frac{\sqrt{3}\pi}{10}$

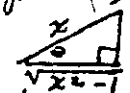
C 4. $-5\sin(\pi^2x^2)\cdot 2\pi^2 x = -10\pi^2 x\sin(\pi^2x^2)$

D 5. $2\cos x + 2\sin 2x = 0 \quad 2\cos x + 4\sin x\cos x = 0 \quad 2\cos x(1+2\sin x)=0$

$x = \pi/2,\ x = 3\pi/2,\ x = \frac{7\pi}{6},\ x = \frac{11\pi}{6}$

$S(0) = -1 \quad S(\frac{\pi}{2}) = 3 \quad S(\frac{7\pi}{6}) = -\frac{1}{2} \quad S(\frac{3\pi}{2}) = -1 \quad S(\frac{11\pi}{6}) = -\frac{3}{2} \quad S(2\pi) = -1$ $[-3/2, 3)$

D 6. $y = 3\ln x \quad y' = \frac{3}{x} \quad m_{tan} = \frac{3}{3e^2} = \frac{1}{e^2} \quad m_{norm} = -e^2$

C 7.  $\cos\theta = \frac{\sqrt{x^2-1}}{x} \quad x \ge 1$ or $x \le -1$

E 8. $\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 3e^t \quad \frac{dy}{dx} = \frac{3e^t}{3t^2} = \frac{e^t}{t^2}$

C 9. $y = \log_3 x \quad dy = \frac{1}{(\ln 3)x}dx \quad$ If $x=c,\ y=d \quad dx=1;\ dy = \frac{1}{(\ln 3)c}$

If $x = c+1,\ y = \frac{1}{(\ln 3)c} + d = \frac{1+(\ln 3)cd}{(\ln 3)c}$

C 10. $\frac{dy}{dx} = \frac{x}{y^2} \quad y^2dy = x\,dx \quad \frac{y^3}{3} = \frac{x^2}{2} + C \quad 9 = 2+c \quad c = 7$

$\frac{y^3}{3} = \frac{x^2}{2} + 7 \quad \frac{y^3}{3} = 18+7 \quad y^3 = 75 \quad y = \sqrt[3]{75}$

B 11. $y' = -6(5x+1)^{-4}\cdot 5 = -30(5x+1)^{-4} = \frac{-30}{(5x+1)^4}$

D 12. $\cos(x+y)[1+y'] = 1 \quad 1+y' = \sec(x+y) \quad y' = \sec(x+y)-1$

$y'' = \sec(x+y)\tan(x+y)[1+y'] \quad y'' = \sec^2(x+y)\tan(x+y)$

$y'' = \sec(x+y)\tan(x+y)(\sec(x+y))$

B 13. $1/2$

A 14. $\frac{1}{2} = e^{1600r} \quad \ln(.5) = 1600r \quad \frac{\ln(.5)}{1600} = r \quad 20 = 50e^{\frac{\ln(.5)}{1600}t} \quad (.4) = e^{\frac{\ln(.5)}{1600}t} \quad \frac{1600\ln(.4)}{\ln(.5)} = t$

A 15. $y' = 2e^{2x} \quad (x, e^{2x}) \quad (0,0) \quad \frac{e^{2x}}{x} = 2e^{2x} \quad 2 = \frac{1}{x} \quad x = \frac{1}{2} \quad (\frac{1}{2}, e)$

C 16. $y' = 5x^4 - 15x^2 \quad y'' = 20x^3 - 30x = 10x(2x^2-3) \quad x = 0 \quad x^2 = 3/2 \quad x = \pm\sqrt{\frac{3}{2}} \quad x = \frac{\pm\sqrt{6}}{2}$

D 17. $S(x) = 2x(4-x^2) = 8x - 2x^3 \quad S'(x) = 8-6x^2 \quad 6x^2 = 8 \quad x^2 = 4/3 \quad x = \frac{2}{\sqrt{3}} \quad L = \frac{4}{\sqrt{3}} \quad W = 4 - \frac{4}{3} = \frac{8}{3} \quad P = \frac{8}{\sqrt{3}} + \frac{16}{3}$

C 18. $y' = 2^{x^2}\cdot\ln 2\cdot 2x = 16(4)(\ln 2) = 64\ln 2$

D 19. $y' = \frac{1}{\sqrt{1-(x+1)^2}} = \frac{1}{\sqrt{-x^2-2x}} \quad m_{tan} = \frac{1}{\sqrt{-\frac{3}{4}+3}} = \frac{1}{\sqrt{3/4}} = \frac{2}{\sqrt{3}} \quad (-1/2, -\pi/6)$

$2x - \sqrt{3}y = -1 + \frac{\sqrt{3}\pi}{6} \quad 12x - 6\sqrt{3}y = -6 + \sqrt{3}\pi$

B 20. $S(x) = e^{\sin x+\cos x} \quad S'(x) = e^{\sin x+\cos x}(\cos x - \sin x) \quad \sin x = \cos x \quad x = \frac{\pi}{4},\ x = \frac{5\pi}{4}$

B 21. $v(t) = 18t^2 - 6t \quad a(t) = 36t - 6 = 6(6t-1) \quad t > 1/6$

C 22. $\cos x > 1/2 \quad [0, \pi/3) \cup (5\pi/3, 2\pi)$

D 23. $\int\sqrt{\sin x}\cos x\,dx \quad u = \sin x \quad du = \cos x\,dx \quad \int u^{1/2}du = \frac{2}{3}u^{3/2} + C = \frac{2\sqrt{\sin^3 x}}{3} + C$

C 24. $y = x^{\ln x} \quad \ln y = (\ln x)(\ln x) \quad \ln y = (\ln x)^2 \quad \frac{y'}{y} = 2(\ln x)\frac{1}{x} \quad y = e^{\ln e} = e$

$y' = 2(1)\left(\frac{1}{e}\right)\cdot e = 2$

A 25. $(0,y)\ (1,8)\ (x,0) \quad \frac{y-8}{-1} = \frac{8}{1-x} \quad y - xy - 8 + 8x = -8 \quad y(1-x) = -8x \quad y = \frac{8x}{x-1} \quad \sqrt{x^2+y^2} = h$

$h^2 = x^2 + \frac{64x^2}{(x-1)^2} \quad 0 = 2x + \frac{(x-1)^2(128x) - 64x^2(2(x-1))}{(x-1)^4} \quad \frac{-128x}{(x-1)^3} = -2x \quad 64x = (x-1)^3$

$4 = x-1 \quad x = 5, \quad y = 10$

A 26. A E 27. $S(x) = \arctan x \quad S'(x) = \frac{1}{1+x^2}$

D 28. $u = \sqrt{x-2} \quad du = \frac{1}{2\sqrt{x-2}}dx \quad u^2 = x-2 \quad dx = 2\sqrt{x-2}\,du \quad x = u^2+2$

$\int_1^3 (u^2+2)^2\cdot u\cdot 2\cdot u\,du \quad 2\int_1^3 (u^6+2u^4+4u^2)du \quad 2[\frac{1}{7}u^7 + \frac{2}{5}u^5 + \frac{4}{3}u^3]_1^3 = 2[\frac{1}{7}(3^7-1) + \frac{2}{5}(3^5-1) + \frac{4}{3}(3^3-1)]$

B 29. $u = 1-2x \quad du = -2dx \quad -\frac{1}{2}\int_9^b u^{1/2}du = \left|-\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]\right. = -\frac{1}{3}(1-27) = \frac{26}{3}$

C 30. $3+b = c+1 \quad c = x \quad$ at $x=1,\ c=6$

$b-c = -2 \quad b-6 = -2 \quad b = 4$