

Garthwa 92

Q92 Team Solutions

① $A^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$

$AB = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 19 \\ -1 & 7 \end{bmatrix}$

$-3(A+B) = -3 \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -12 & -15 \\ -9 & -12 \end{bmatrix}$ $B^2 = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 11 \end{bmatrix}$

$-A^{-1} + AB - 3(A+B) + B^2 = \begin{bmatrix} -12 & 11 \\ -6 & 1 \end{bmatrix}$

- ② (A) $\frac{523}{705} \quad \frac{1430}{8}$
- (B) $\frac{64\sqrt{3174}}{256} \quad \frac{475}{56}$
- (C) $\frac{10 \text{ or } 110}{2} \quad \frac{2 \text{ or } 3 \text{ or } 6 \text{ or } 8}{8}$

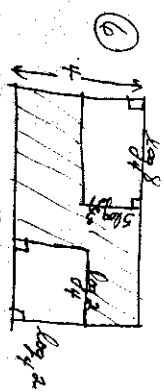
$1430 + 475 + 236 = \boxed{23638}$

③ $f(13) = 7 \cdot 20 = 9a + 3b + c$ $\rightarrow 1 \neq 5a + b$ $1 = 5(-1) + b$
 $f(12) = 16 \cdot 20 = 6 = 4a + 2b + c$ $\rightarrow -1 - 3 = -3a - b$ $b = 6$
 $f(11) = 3 \cdot 20 = 3 = a + b + c$ $\rightarrow -2 = 2a$ $3 = -1 + 6 + c$
 $4a + b + c = 8$ $-1 = a$ $3 = 5 + c$
 $-1 + 6 - 2 = 3$ or just use the last equation and got it immediately!
 $-2 = c$

④ $4(x^2 - 6x + 9) + 9(y^2 + 4y + 4) = -36 + 72$
 $4(x-3)^2 + 9(y+2)^2 = 36$ $C(3, -2)$
 $x^2 - 8x + 16 + y^2 + 4y + 4 = -18 + 20$
 $(x-4)^2 + (y+2)^2 = 2$
 $C(4, -2)$

⑤ $y(12) = \frac{4-6-2}{18-5} = \frac{-4}{13}$ $\frac{4\sqrt{5}}{5} = 11$
 $C = 3 \cdot \frac{4}{13} (3 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3}) = 3 + 3 = 1 + 3$
 $D = 3 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = 1 + \frac{2}{3}$
 $3 \cdot \frac{1}{3} + 1 = 1 + \frac{2}{3}$

$ABCD = -\frac{4\sqrt{5}}{13} \cdot 20 \cdot 4 \cdot 3 = \boxed{-320\sqrt{5}}$



⑥ $20 - \frac{3}{2} \cdot \frac{5}{3} - \frac{1}{2} \cdot \frac{1}{2} = 20 - \frac{5}{2} - \frac{1}{4} = \frac{80}{4} - \frac{10}{4} - \frac{1}{4} = \frac{69}{4}$ or $17\frac{1}{4}$

⑦ $(x^2 - 1)^2 = 5 - 4(x^2 - 1)$

$y^2 + 4y - 5 = 0$
 $(y+5)(y-1) = 0$
 $y = -5$ or $y = 1$
 $x^2 - 1 = -5$ or $x^2 - 1 = 1$
 $x^2 = -4$ or $x^2 = 2$
 $x = \pm 2i$ or $x = \pm \sqrt{2}$
 $A - B = -5536$
 $B = \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 5} (2x)^3 y^3 = 20 \cdot 8x^3 y^3 = 160$

⑧ $\frac{8}{B} \cdot \frac{6}{B} \cdot \frac{7}{B} \cdot \frac{5}{B} \cdot \frac{4}{B} \cdot \frac{5}{B} = \frac{1201600}{B^6}$

⑨ $2(n-1) = 43(4n) = \boxed{903}$

⑩

Tanya's Age	Now	10 Years Ago	Yes
$3x$	$3x+10$	$3x+10$	Yes
x	$x+4$	$x+10$	Yes

$3x + 10 = 2(x + 10)$
 $x = 10$

Tanya's age now is $\boxed{34}$ = T
 $T - D = 34 - 18 = \boxed{16}$

⑪ $5d = 90$ $d = 18$ $(x^2 - 3)(x^2 - 2x - 3)$
 $10d + 5n = 290$
 $5d + 5n = 200$
 $5d = 90$
 $x = \frac{y+5}{2}$ $y = \frac{x^3 - 4x^2 + x + 6}{x-2}$ $x \neq 2$
 $y = x^2 - 2x - 3$
 $y = (y+5)^2 - 2(y+5) - 3$
 $y = y^2 + 10y + 25 - 2y - 10 - 3$
 $0 = y^2 + 7y + 12$
 $(y+3)(y+4) = 0$
 $y = -3$ or $y = -4$
 $x = -3 + 5 = 2$ doesn't work
 $x = -4 + 5 = 1$
 $(1, -4)$

⑫ $x = \frac{y+5}{2}$ $y = \frac{x^3 - 4x^2 + x + 6}{x-2}$ $x \neq 2$
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$$(13) \quad 3^{x+1} + 3^{-x} = 4$$

$$3 \cdot 3^x + \frac{1}{3^x} = 4$$

$$3y + \frac{1}{y} = 4$$

$$3y^2 + 1 = 4y$$

$$3y^2 - 4y + 1 = 0$$

$$(3y-1)(y-1) = 0$$

$$y = \frac{1}{3} \text{ or } y = 1$$

$$3^x = \frac{1}{3} \text{ or } 3^x = 1$$

$$\boxed{X = -1 \text{ or } X = 0}$$

$$(14) \quad A = 10,800$$

$$= 2^4 \cdot 3^3 \cdot 5^2$$

$$A = 5 \cdot 4 \cdot 3 = 60$$

$$B = 2^5 \cdot 3^2 \cdot 5^3$$

use all factors except 2°

$$5 \cdot 3 \cdot 4 = 60$$

$$\frac{A}{B} = \frac{60}{60} = \boxed{1}$$

$$(15) \quad 3 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3 + \dots}}}}$$

$$X = 3 + \frac{1}{2 + \frac{1}{X}}$$

$$X = \frac{3}{1} + \frac{X}{2X+1}$$

$$\frac{X}{1} = \frac{6X+3+X}{2X+1}$$

$$2X^2 + X = 7X + 3$$

$$2X^2 - 6X - 3 = 0$$

$$X = \frac{6 \pm \sqrt{36 - 4(2)(-3)}}{4}$$

$$= \frac{6 \pm \sqrt{36 + 24}}{4}$$

$$= \frac{6 \pm \sqrt{60}}{4}$$

$$= \frac{6 \pm 2\sqrt{15}}{4}$$

$$\boxed{X = \frac{3 \pm \sqrt{15}}{2}}$$

$$\downarrow$$

$$\boxed{\frac{3 + \sqrt{15}}{2}} \text{ only pos. sol.}$$