

10

$$\begin{aligned}
 1) \quad A &= \cos 75^\circ = \frac{\sqrt{6}-\sqrt{2}}{4} \\
 B &= \sin 105^\circ = \frac{\sqrt{6}+\sqrt{2}}{4} \\
 C &= \sin 270^\circ = -1 \\
 D &= \cos 72^\circ = \frac{\sqrt{5}-1}{4} \\
 ABCD &= \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)(-1)\left(\frac{\sqrt{5}-1}{4}\right) \\
 &= \frac{1}{4}\left(\frac{1-\sqrt{5}}{4}\right) \\
 &= \frac{1-\sqrt{5}}{16}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad W &= \tan a + \tan b \\
 X &= \tan a - \tan b \\
 Y &= \cot a + \cot b \\
 Z &= \cot a - \cot b
 \end{aligned}$$

$$\begin{aligned}
 W &= \tan a + \tan b = \frac{\sin(a+b)}{\cos a \cos b} \\
 X &= \tan a - \tan b = \frac{\sin(a-b)}{\cos a \cos b} \\
 Y &= \cot a + \cot b = \frac{\sin(a+b)}{\sin a \sin b} \\
 Z &= \cot a - \cot b = \frac{\sin(b-a)}{\sin a \sin b}
 \end{aligned}$$

$$\begin{aligned}
 \frac{W}{X} \cdot \frac{Z}{Y} &= \frac{\sin(a+b)}{\sin(a-b)} \cdot \frac{\sin(b-a)}{\sin(a+b)} \\
 &= -\frac{\sin(a-b)}{\sin(a-b)} \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \frac{2}{17} &= \frac{a+b}{ab} \rightarrow 17a+17b=2ab \\
 &\rightarrow a(2b-17)=17b \\
 &\rightarrow a = \frac{17b}{2b-17}
 \end{aligned}$$

set b so denominator = 1  $\rightarrow b=9, a=153$   
 $a+b = 9+153 = 162$

$$\begin{aligned}
 4) \quad \vec{u} &= (7, -2, 2) \quad \vec{v} = (-1, 1, 5) \\
 A &= \vec{u} \cdot \vec{v} \\
 B &= \vec{u} \times \vec{v} \quad \text{find } A \cdot B \\
 A &= (7, -2, 2) \cdot (-1, 1, 5) = -7 - 2 + 10 = 1 \\
 B &= (7, -2, 2) \times (-1, 1, 5) = (-12, -37, 5) \\
 AB &= (-12, -37, 5)
 \end{aligned}$$

Precalc Team Solutions - 3/14/98 - F.A. 11/1/98

$$\begin{aligned}
 2) \quad \sin[2 \arccos(-\frac{3}{5})] &= \\
 t &= \arccos(-\frac{3}{5}) \rightarrow \cos t = -\frac{3}{5} \\
 &\rightarrow \sin t = \frac{4}{5} \\
 \sin 2t &= 2 \sin t \cos t \\
 &= 2(-\frac{3}{5})(\frac{4}{5}) \\
 &= -\frac{24}{25}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad BC &= 2.3, AC = 4.5, \angle A = 42^\circ \\
 \text{Law of sines: } \frac{2.3}{\sin 42^\circ} &= \frac{4.5}{\sin B} \\
 \Rightarrow \sin B &= 1.309 \\
 \Rightarrow \text{No Sol.}
 \end{aligned}$$

$$\begin{aligned}
 6) \quad A &= 1, B = 1, C = 0, D = e \\
 ABCD &= 0
 \end{aligned}$$

$$\begin{aligned}
 7) \quad (4x^3 - \frac{1}{2}x^{-2})^{10} \\
 {}_{10}C_4 (4x^3)^4 \left(-\frac{1}{2}x^{-2}\right)^6 &= 210(2^8 x^{12})(2^{-6} x^{-12}) \\
 &= 840
 \end{aligned}$$

$$\begin{aligned}
 8) \quad A &= x, B = x, C = 0, D = x \\
 ABCD &= 0
 \end{aligned}$$

$$\begin{aligned}
 9) \quad \text{If } f(x) &= x^2 - 5, x \geq 0 \\
 \text{domain of } f^{-1} &\text{ same as range of } f \\
 \Rightarrow [5, \infty)
 \end{aligned}$$

$$\begin{aligned}
 10) \quad 9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) &= 11 + 9 + 16 = 36 \\
 9(x-1)^2 + 4(y+2)^2 &= 36 \\
 \frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} &= 1
 \end{aligned}$$

$$\begin{aligned}
 (A, B) &= (1, -2) \\
 C &= \sqrt{9-4} = \sqrt{5} \\
 D &= \frac{2 \cdot 9}{2} = 9
 \end{aligned}$$

$$ABCD = -18\sqrt{5}$$

$$\begin{aligned}
 11) \quad |2x-3| &= |7-3x| \\
 2x-3 &= 7-3x \quad 2x-3 = -(7-3x) \\
 x &= 2 \quad x = 4 \\
 \text{Sum} &= 2+4 = 6
 \end{aligned}$$

$$\begin{aligned}
 12) \quad y &= -2(x^2 - 4x + 4) + 3 \\
 &= -2(x-2)^2 + 3 \\
 \text{parabola opening downward,} \\
 \text{max value at } y &= 3.
 \end{aligned}$$

$$\begin{aligned}
 13) \quad P(x) &\text{ has two variations in} \\
 &\text{sign, therefore number} \\
 &\text{of positive roots is two or zero} \\
 \{0, 2\}
 \end{aligned}$$

$$14) \quad {}_{10}C_6 (2u^9)^4 \left(-\frac{1}{4}v^4\right)^6 = \frac{105}{128} u^{12} v^{24}$$