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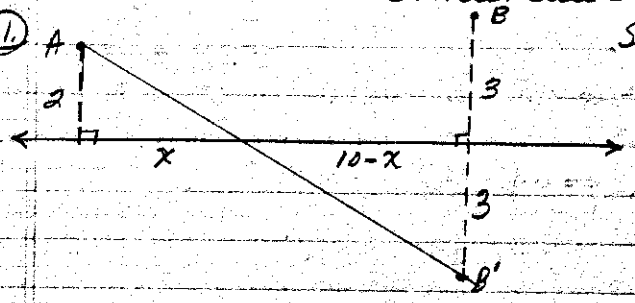
# A.P. Leto Comprehensive High School
# Mu Alpha Theta Invitational Competition
# Saturday, February 2, 1991

## Individual Test Answers

| Algebra I | Geometry | Algebra II | Pre-Calculus | Calculus |
|---|---|---|---|---|
| 1) A | 1) A | 1) C | 1) D | 1) B |
| 2) D | 2) B | 2) C | 2) B | 2) B |
| 3) C | 3) A | 3) D | 3) A | 3) E |
| 4) B | 4) B | 4) A | 4) D | 4) D |
| 5) A | 5) C | 5) B | 5) C | 5) B |
| 6) B | 6) B | 6) E | 6) B | 6) D |
| 7) C | 7) E | 7) E | 7) A | 7) D |
| 8) A | 8) D | 8) D | 8) C | 8) A |
| 9) D | 9) B | 9) B | 9) D | 9) C |
| 10) D | 10) B | 10) B | 10) C | 10) C |
| 11) A | 11) D | 11) E | 11) B | 11) C |
| 12) E | 12) D | 12) C | 12) C | 12) C |
| 13) C | 13) B | 13) C | 13) A | 13) D |
| 14) B | 14) C | 14) B | 14) D | 14) A |
| 15) A | 15) D | 15) C | 15) A | 15) A |
| 16) B | 16) D | 16) E | 16) A | 16) C |
| 17) B | 17) D | 17) C | 17) D | 17) D |
| 18) A | 18) C | 18) A | 18) C | 18) E |
| 19) D | 19) D | 19) C | 19) B | 19) A |
| 20) C | 20) D | 20) C | 20) A | 20) C |
| 21) C | 21) D | 21) B | 21) C | 21) B |
| 22) B | 22) D | 22) C | 22) B | 22) D |
| 23) C | 23) C | 23) A | 23) D | 23) A |
| 24) A | 24) A | 24) D | 24) E | 24) C |
| 25) C | 25) A | 25) A | 25) C | 25) C |
| 26) D | 26) B | 26) A | 26) B | 26) E |
| 27) B | 27) B | 27) B | 27) C | 27) A |
| 28) A | 28) D | 28) D | 28) D | 28) B |
| 29) E | 29) E | 29) D | 29) A | 29) D |
| 30) C | 30) E | 30) E | 30) C | 30) B |

Leto 91

Pre-Calculus  Individual Solutions

① 


Similar Δ's so –

$\frac{2}{x} = \frac{3}{10-x}$

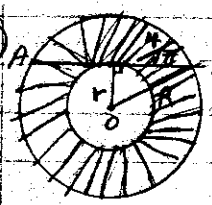
$20-2x = 3x$

$20 = 5x$

$4 = x$

∴ D

② $S = \log \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdots \frac{99}{100} = \log \frac{1}{100} = -2$

③ 

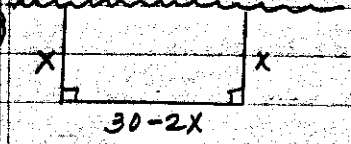

R – radius of Big circle

r – radius of small circle

$R^2 - r^2 = \frac{16}{\pi}$

$A = (R^2 - r^2)\cdot\pi$

$\frac{16}{\pi}\cdot\pi = 16$ ∴ A

④ 


$x(30-2x) = 108$

$2x^2 - 30x - 108 = 0$

$(2x-12)(x-9) = 0$

$x = 6$ or (9) largest ∴ D

⑤ All factors of 5 will make a zero with 2. There's more 2's than 5's so count the 5's.

$5\overline{)127}\ 25$

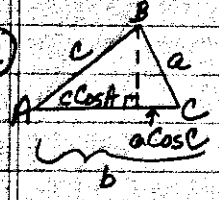
$25\overline{)127}\ 5$

$125\overline{)127}\ 1$

$25+5+1 = (31)$ ∴ C

⑥ $2^{30} + 3\cdot 3^8 2^{20} + 3\cdot 2^{10}\cdot 3^{16} + 3^{24}$ Now if you let $x = 2^{10}$ and $y = 3^8$ this is $(x+y)^3$

Thus $\sqrt[3]{(2^{10}+3^8)^3} = 2^{10} + 3^8 = 1024 + 6561 = 7585$ ∴ B

⑦ 


$\frac{\cos A}{a} + \frac{\cos C}{c} = \frac{c\cos A + a\cos C}{ac} = \frac{b}{2b} = \frac{1}{2}$

∴ A

(8) $6^x + 6^{1-x} = 7$

$6^x + 6 \cdot 6^{-x} = 7$

Let $y = 6^x$

$y^2 - 7y + 6 = 0$

$(y-6)(y-1) = 0$

$y = 6$ or $y = 1$ so

$6^x = 6$ or $6^x = 1$

$x = 1$ or $0$

$\{0, 1\}$

$\therefore C$

(9) $\frac{1}{(\sec x + \tan x)} \cdot \frac{(\sec x - \tan x)}{(\sec x - \tan x)} = \frac{\sec x - \tan x}{\sec^2 x - \tan^2 x} = \frac{\sec x - \tan x}{1}$

$\therefore D$

(10) $(-x + 3y = -4) \times 2$

$2x - 4y = 5$

$-2x + 6y = -8$

$2y = -3$

$y = -3/2$

$-x + 3(-3/2) = -4$

$-x - 9/2 = -4$

$-x = +1/2$

$x = -1/2$

$\therefore -1/2\, a - 3/2\, b$

$C$

(11) Use 3 sol. in $f(x) = ax^2 + bx + c$.

$f(1) = a + b + c = 0$

$f(-1) = (a - b + c = -2) \times 2$

$f(2) = 4a + 2b + c = 7$

$2a - 2b + 2c = -4$

$6a + 3c = 3$

$-3(2a + 2c = -2)$ Add Top 2

$-6a - 6c = 6$

$-3c = 9$

$c = -3$

$6a + 3(-3) = 3$

$6a = 12$

$a = 2$

$2 + b - 3 = 0$

$b = 1$

So $f(x) = 2x^2 + x - 3$

and $f(3) = 2 \cdot 9 + 3 - 3 = 18$

$\therefore B$

(12) $\sin 20° + \sin 40° = 2 \sin 30° \cos 10° = \cos 10° = \sin 80°$ $\therefore C$

(13) $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \ldots$

$d = 1/6$ So $a_{11} = 1/3 + 10(1/6)$

$= 1/3 + 5/3 = 2$ $\therefore$ The 11th harmonic is $1/2$ A

(8)

(14) Use $\tan(A+B)$ where $A = \text{Arctan } 2x$, $B = \text{Arctan } 3x$

$\tan(A+B) = \tan \pi/4$

$\frac{2x+3x}{1-6x^2} = 1$

$1-6x^2 = 5x$

$6x^2+5x-1=0$

$(6x-1)(x+1)=0$

$x = 1/6 \therefore D$

(15) $\log_3 x + \log_9 x + \log_{81} x = 7$

$\log_3 x + \frac{1}{2}\log_3 x + \frac{1}{4}\log_3 x = 7$

$(1+1/2+1/4)\log_3 x = 7$

$7/4 \log_3 x = 7$

$\log_3 x = 4$

$x = 3^4 = 81 \therefore A$

(16) $\cos 2x = 1 - 2\sin^2 x$

$1 - 2 \cdot 9/16 = 1 - 18/16 = -1/8 \therefore A$

(17) $2x - 5y = 8$
$7y + 2z = -7$ } Just add

$2x + 2y + 2z = 1$

$x+y+z = 1/2$. So $(x+y+z)^2 = 1/4 \therefore D$

(18)


$\left(\frac{5}{2}x\right)^2 + (6x)^2 = (26)^2$

$\frac{25}{4}x^2 + 36x^2 = 676$

$\frac{169}{4}x^2 = 676$

$x^2 = 676 \cdot \frac{4}{169}$

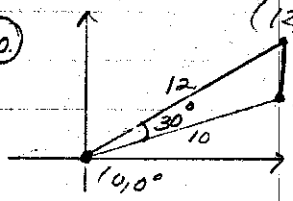
$x = \frac{26 \cdot 2}{13} = 4 \therefore AB = 20$ C

(19) $\frac{\log 25}{\log a} = \frac{\frac{2}{3}\log 25}{\log b}$ $\quad \log_a b = \frac{2}{3}$

$\frac{\log b}{\log a} = \frac{2}{3}$ $\quad a^{2/3} = b$

$a = b^{3/2}$ ∴ B

(20)




$K = \frac{1}{2} \cdot 10 \cdot 12 \cdot \sin 30°$

$5 \cdot 12 \cdot \frac{1}{2} = 30$ ∴ A

(21) $\frac{8!}{2!\,5!\,1!} = \frac{8 \cdot 7 \cdot 6}{2} = 168(2)^2(-1)^5(2)$

$168(-8) = -1344$ ∴ A

(22) V of cylinder $= \pi r^2 h = 30$ $\quad h = 2r$

$= \pi r^2 \cdot 2r = 30$

$r^3 = \frac{15}{\pi}$

V of sphere $= \frac{4}{3}\pi r^3$ 5

$\frac{4}{3}\pi \cdot \frac{15}{\pi} = 20$ ∴ B

(23) $(1-i)^{11} = [\sqrt{2}\,\mathrm{cis}(-45°)]^{11}$

$32\sqrt{2}\,\mathrm{cis}(-495°)$

$32\sqrt{2}\,\mathrm{cis}\,225 = 32\sqrt{2}\left(-\frac{1}{\sqrt{2}} + -\frac{1}{\sqrt{2}}i\right)$

$\boxed{-32 - 32i}$ ∴ D

(24) $\div$ by 4 $\quad x^2 + y^2 + 2x + 4y + 7 = 0$

$x^2 + 2x + 1 + y^2 + 4y + 4 = -7 + 5$

(-2) No graph

∴ E

(25) $r = \frac{1}{1 + \sin\theta}$

$r + r\sin\theta = 1$

$r + y = 1$

$r = 1 - y$

$r^2 = 1 - 2y + y^2$

$x^2 + y^2 = 1 - 2y + y^2$ ∴ C

(26) $(p \wedge \sim q) \vee [\sim p \wedge (q \vee \sim q)] = (p \wedge \sim q) \vee (\sim p \vee t)$

$= (p \wedge \sim q) \vee \sim p$

$= (p \vee \sim p) \wedge (\sim q \vee \sim p)$

$= t \wedge (\sim q \vee \sim p)$

$= \sim q \vee \sim p$

$= q \rightarrow \sim p$

(27) $ab(c+c') + ab'c = ab + ab'c$

$= a(b + b'c)$

$= a(b+b')(b+c) = a(b+c) \therefore C$

(28)

| | | | |
|---|---|---|---|
| -2 | 1 | -2 | 1 | -3 |
| | | -2 | 8 | -18 |
| -2 | 1 | -4 | 9 | -21 |
| | | -2 | 12 | |
| -2 | 1 | -6 | 21 | |
| | | -2 | | |
| | 1 | -8 | | |

$\therefore x^3 - 8x^2 + 21x - 21 = 0$

$1 - 8 + 21 - 21 = -7 \therefore D$

(29) $n^2 + 2n + 3 = 3(3n+7)$

$n^2 - 7n - 18 = 0$

$(n-9)(n+2) = 0$

$n = 9 \therefore A$

(30) $\begin{bmatrix} 0 & -3 \\ 2 & 4 \end{bmatrix} X = \begin{bmatrix} 0 & -18 \\ 10 & 28 \end{bmatrix}$

$X = \frac{1}{6} \begin{bmatrix} 4 & 3 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -18 \\ 10 & 28 \end{bmatrix}$

$X = \frac{1}{6} \begin{bmatrix} 30 & 12 \\ 0 & 36 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 0 & 6 \end{bmatrix} \therefore C$