A.P. Leto Comprehensive High School
Mu Alpha Theta Invitational Competition
Saturday, February 2, 1991

Team Round Answers

Algebra I

1) Quadrant I
2) 864
3) 47
4) 370.5
5) $\frac{6}{5}$
6) 2
7) 2
8) 29
9) $x + y - 9 = 0$
10) {2, 0, -1}
11) $\frac{47}{36}$ or $1\frac{11}{36}$
12) 21
13) 45,369
14) $\frac{7}{2}$
15) -12

Geometry

1) 84
2) 118
3) $\frac{21\pi}{4}$
4) 12
5) 60 meters
6) $\frac{16}{3}$
7) $2a^2$
8) $\sqrt{97}$
9) $\frac{8}{7}$
10) 112
11) 72
12) 12
13) 8 feet
14) $\frac{4}{5}$
15) $100\pi - 128$

Algebra II

1) 121
2) $14\sqrt{2}$
3) 14
4) $\{-2, 3, \frac{-1 \pm i\sqrt{3}}{2}\}$
5) 12
6) $-3 \leq x < -1$
7) 1,489
8) 480
9) 30
10) $\{-3 \leq x \leq 2$ or $x \geq 4\}$
11) $\frac{16\pi}{3}$
12) 20
13) 20
14) 644
15) 16

Pre-Calculus

1) {-3, 1}
2) 8 ft
3) $\frac{-3}{2}$
4) 1
5) $\frac{2\sqrt[3]{4} + 6}{31}$
6) 4
7) $\frac{14}{3} + 2\sqrt{5}$
8) 6
9) 135
10) $4 + 4\sqrt{13}$
11) $\frac{-15}{4}$
12) 6,479
13) 17,550
14) $\frac{37\sqrt{7}}{4}$
15) -14

Calculus

1) -6
2) 3
3) $\frac{9}{4}$
4) 6
5) $\frac{-3\sqrt{3}}{4}$
6) $4x+48y+47\pi-384=0$
7) $\frac{-80e^9}{7}$
8) $\frac{1}{16}$
9) $x - y - 1 = 0$
10) $\frac{-\sqrt{3}}{3e}$
11) $2x+3y+2\sqrt{5}=0$
$2x + 3y - 2\sqrt{5} = 0$
12) 50
13) $3 + 4\ln 2$
or $3 + 2\ln 4$
14) $a = 2; b = -1$
15) $2 \times 2 \times \frac{5}{2}$

Calculus Team

1. $y = x^4 + bx^2 + 8x + 1 \quad y' = 4x^3 + 2bx + 8 \quad y'' = 12x^2 + 2b$

$0 = 4x^3 + 2bx + 8$ when $8 = 12x^2 + 2b \quad 2b = -12x^2 \quad b = -6x^2$

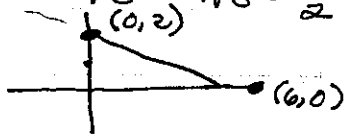
$0 = 4x^3 - 12x^3 + 8 \quad 8x^3 = 8 \quad x = 1 \quad \boxed{b = -6}$

2. $\lim_{x\to 0} \frac{x(\cos x - 1)}{\sin x - x} = \lim_{x\to 0} \frac{-x\sin x + (\cos x - 1)}{\cos x - 1} = \lim_{x\to 0} \frac{-x\cos x - \sin x - \sin x}{-\sin x}$

$\lim_{x\to 0}$ ~~$\frac{x\sin x\cos x}{\sin x}$~~ $\lim_{x\to 0} \frac{x\sin x - \cos x - \cos x - \cos x}{-\cos x} = \boxed{3}$

3. $\frac{5(4) - 5(1)}{3} = 1 - \frac{1}{\sqrt{t}} \quad \frac{0+1}{3} = 1 - \frac{1}{\sqrt{t}} \quad -\frac{2}{3} = -\frac{1}{\sqrt{t}} \quad 2\sqrt{t} = 3 \quad \sqrt{t} = \frac{3}{2} \quad \boxed{t = \frac{9}{4}}$

4. $y = \frac{3}{x} \quad y' = -3x^{-2} \quad m_{tan} = -\frac{1}{3} \quad x + 3y = 6$



$\frac{1}{2}(6)(2) = \boxed{6}$

5. $y = 2\sqrt{3}\sin x + 6\cos x \quad y' = 2\sqrt{3}\cos x - 6\sin x \quad 6\sin x = 2\sqrt{3}\cos x$

$\tan x = \frac{\sqrt{3}}{3} \quad x = \frac{\pi}{6} \quad 2\sqrt{3}\left(\frac{1}{2}\right) + 6\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3} + 3\sqrt{3} = 4\sqrt{3}$ or

$\sqrt{(2\sqrt{3})^2 + (6)^2} = \sqrt{12 + 36} = \sqrt{48} = 4\sqrt{3} = A$

$B = \lim_{h\to 0} -3(2+h)^{-4} = \frac{-3}{16} \quad -\frac{3}{16} \cdot \frac{4\sqrt{3}}{1} = \boxed{\frac{-3\sqrt{3}}{4}}$

6. $y = 4\sec^2 x - 4x \quad y' = 8\sec^2 x \tan x - 4$ at $x = \frac{\pi}{4}$, $y = 8 - \pi$ $\quad 4(2) - \pi$

$m_{tan} = 8(2)(1) - 4 = 12 \quad m_{norm} = -\frac{1}{12} \quad x + 12y = \frac{\pi}{4} + 12(8 - \pi)$

$x + 12y = \frac{\pi}{4} + 96 - 12\pi$

$4x + 48y = \pi + 384 - 48\pi$

$\boxed{4x + 48y + 47\pi - 384 = 0}$

7. $A = -\frac{5}{7} \quad B = 32e^8 \quad S'(x) = 4e^{2x^2} \cdot 4x \quad S'(2) = 32e^8$

$S(x) = \ln 4x \quad x = \ln 4y \quad e^x = 4y \quad S^{-1}(x) = \frac{1}{4}e^x \quad C = \frac{e}{4}$

$S'(x) = 3x^2 - 3 \quad S(0) = 0 \quad S(1) = -2 \quad S(2) = 2 \quad D = 2$

$ABCD = -\frac{5}{7} \cdot \frac{32e^8}{1} \cdot \frac{e}{4} \cdot \frac{2}{1} = \boxed{\frac{-80e^9}{7}}$

8. $a_1 = \sqrt{2 + \sqrt{x}} \quad \lim_{x\to 4} \frac{\sqrt{2+\sqrt{x}} - 2}{x - 4} = \lim_{x\to 4} \frac{1}{2}(2 + \sqrt{x})^{-\frac{1}{2}} \cdot \frac{1}{2\sqrt{x}}$

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} = \boxed{\frac{1}{16}}$

9. $y = \ln x \quad y = x\ln y$

$\frac{1}{x} = 1 + \ln x$ at $x = 1$. $m_{tan} = 1$ point $(1, 0)$

$x - y = 1 \quad \boxed{x - y - 1 = 0}$

10. $A=\frac{-1}{\sqrt{3}}$ $f(x)=\arccos\frac{x}{2}$ $\frac{-1}{\sqrt{1-\frac{x^2}{4}}}\cdot\frac{1}{2}$ $f'(1)=\frac{-1}{\sqrt{\frac{3}{4}}}\cdot\frac{1}{2}=\frac{-2}{\sqrt{3}}\cdot\frac{1}{2}$

$B=\frac{-1}{e}$ $y=xe^x$ $y'=xe^x+e^x=e^x(x+1)$ at $x=-1$ $y=\frac{-1}{e}$

$C=-1$ $y=\frac{x^3}{3}+x^2-3x+7$ $-4=x^2+2x-3$ $x^2+2x+1=0$ $(x+1)^2=0$ $x=-1$

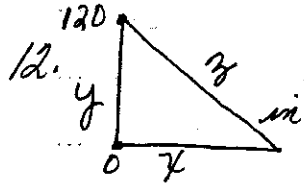
$ABC=\frac{-\sqrt{3}\cdot}{3e}$

11. $x^2+3xy+y^2=5$ at $y=0$ $x^2=5$ $x=\pm\sqrt{5}$

$2x+3xy'+3y+2yy'=0$ $(3x+2y)y'=-2x-3y$ $y'=\frac{-2x-3y}{3x+2y}$

at $y=0$ $y'=-\frac{2}{3}$ $2x+3y=2\sqrt{5}$ $2x+3y-2\sqrt{5}=0$

$2x+3y=-2\sqrt{5}$ $2x+3y+2\sqrt{5}=0$

12. 

$z^2=x^2+y^2$ in 2 seconds $2z\left(\frac{dz}{dt}\right)=2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)$

$y=60$, $x=80$, $z=100$ $100\left(\frac{dz}{dt}\right)=80(40)+60(30)$

$\frac{dz}{dt}=32+18=50$

13. $a(t)=(t+1)^2\ln(t+1)$ $v(t)=(t+1)^2\left(\frac{1}{t+1}\right)+[\ln(t+1)]\,2(t+1)]$

$v(t)=(t+1)+\ln(t+1)\cdot 2(t+1)=(t+1)[1+2\ln(t+1)]$

$a(t)=(t+1)\cdot\frac{2}{(t+1)}+[1+2\ln(t+1)]=3+2\ln(t+1)$

$a(3)=3+2\ln 4$ or $3+4\ln 2$

14. at $x=1$ $a+b=1$ $2x=a$ $a=2$ $b=-1$

15. $10=x^2h$ $15x^2+4(xh)=V$ $V(x)=15x^2+4x\left(\frac{10}{x^2}\right)$

$\frac{10}{x^2}=h$ $V(x)=15x^2+40x^{-1}$ $V'(x)=30x-40x^{-2}$

$30x=\frac{40}{x^2}$ $x^3=8$ $x=2$ $2\times 2\times\frac{5}{2}$