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A.P. Leto Comprehensive High School
 Mu Alpha Theta Invitational Competition
 Saturday, February 2, 1991

Algebra II Individual

1. Let $z = 2 + i$. If $z^2 + (z)(\bar{z}) = a + bi$, where \bar{z} is the conjugate of z and $i = \sqrt{-1}$, then the value of b is

- a) $4i$ b) 2 c) 4 d) 8 e) NOTA

2. The numerator of a fraction is one more than triple the denominator. Adding 4 to each makes the numerator double the denominator. Find the numerator.

- a) 3 b) 7 c) 10 d) 14 e) NOTA

3. For all non-zero numbers x and y such that $x = \frac{1}{y}$, $(x - \frac{1}{x})(y + \frac{1}{y}) = ?$

- a) $2x^2$ b) $2y^2$ c) $x^2 + y^2$ d) $x^2 - y^2$ e) NOTA

4. If $2x + y = 1$ and $3x - 2y = 8$, then $2x^2 + 3xy + y^2 = ?$

- a) $\frac{-3}{7}$ b) $\frac{-51}{49}$ c) $\frac{1179}{49}$ d) $\frac{23}{7}$ e) NOTA

5. Simplify: $\frac{2b(12 - 14)}{20 + 3(-2)} + \frac{6b(15 - 11) + 6 - (-2)}{2^2 + 3(2)}$

- a) $\frac{4b + 1}{35}$ b) $\frac{4b + 7}{35}$ c) $\frac{4b + 14}{35}$ d) $\frac{b}{70}$ e) NOTA

6. The letters in the word "GREATER" are listed in a row. What is the probability that three consecutive letters are consonants?

- a) $\frac{3}{4}$ b) $\frac{4}{35}$ c) $\frac{4}{7}$ d) $\frac{5}{7}$ e) NOTA

7. Simplify: $\frac{\frac{a}{b} + 2 + \frac{b}{a}}{a^2 - b^2}$

- a) $a - b$ b) $\frac{a - b}{a + b}$ c) $\frac{a + b}{ab}$ d) $\frac{a - b}{ab}$ e) NOTA

8. Solve for y : $5 < |2y - 3| < 7$

- a) $4 < y < 5$ b) $-2 < y < 1$ or $4 < y < 5$
 c) $-5 < y < -4$ or $4 < y < 5$ d) $-2 < y < -1$ or $4 < y < 5$
 e) NOTA

9. Solve for x : $\sqrt{5x-1} = 5 - \sqrt{4x-4}$

- a) $\{42 + 4\sqrt{155}\}$ b) $\{2\}$
 c) $\{2, 442\}$ d) \emptyset
 e) NOTA

10. An airplane flew with the wind for 3 hours. The return trip against the wind took $3\frac{1}{2}$ hours. If the wind velocity was 25 mph, what is the cruising speed (rate in still air) of the plane?

- a) 275 mph b) 325 mph c) 340 mph d) 350 mph e) NOTA

11. A line L contains the points $(2,6)$ and $(1,4)$. The point $(4,a)$ is also on the line. Write the equation of the line perpendicular to L passing through $(4,a)$.

- a) $2x - y = -2$ b) $x + 2y = 14$
 c) $x - 2y = 24$ d) $x - 2y = -16$
 e) NOTA

12. Find the zeroes of $y = \log(x^2 - 5x - 6)$

- a) $\{-1,6\}$ b) $\{\frac{5 + \sqrt{53}}{2}\}$
 c) $\{\frac{5 \pm \sqrt{53}}{2}\}$ d) \emptyset
 e) NOTA

13. Which of the following is evident from the sketch of the graph of $P(x) = x^2(4-x)$?

- a) The curve has three distinct x intercepts.
 b) $P(x) > 0$ for $x > 4$
 c) The curve touches the x -axis at $x = 0$ and crosses the x -axis at $x = 4$.
 d) The curve contains the point $(5, 25)$
 e) NOTA

14. $\frac{(\sqrt[3]{81x^5})(\sqrt[3]{27x^4})}{\sqrt[3]{16x^4}}$

- a) $\frac{3x\sqrt[3]{12x^2}}{2}$ b) $\frac{3x\sqrt[3]{3}}{2\sqrt[3]{2x}}$
 c) $\frac{9x\sqrt[3]{3}}{2\sqrt[3]{2x}}$ d) $\frac{9x\sqrt[3]{12x^2}}{4}$
 e) NOTA

15. In a triangle whose perimeter is 60 centimeters, the length of the longest side is 6 centimeters less than the sum of the lengths of the other sides. Three times the length of the shortest side is 40 centimeters minus the difference between the lengths of the other sides. Find twice the length of the shortest side.

- a) 11.5 b) 21.5 c) 23 d) 46 e) NOTA

16. Find the sixth term in the expansion of $\left(2x^2 - \frac{1}{y\sqrt{2}}\right)^8$.

- a) $\frac{-112(x^6)(\sqrt{2})}{y^5}$ b) $\frac{56(x^5)(\sqrt{2})}{y^5}$
 c) $\frac{-56x^6}{(y^5)\sqrt{2}}$ d) $\frac{56(x^6)(\sqrt{2})}{y^5}$
 e) NOTA

17. Simplify: $\frac{7! + 6! + 5! + 4!}{4!(4! - 4)}$

- a) 1 b) $\frac{49}{4}$ c) $\frac{123}{10}$ d) 41 e) NOTA

18. $\left(\frac{x^2 + 6x + 9}{3 - 2x - x^2}\right)\left(\frac{x^2 + 2x + 3}{x^4 + 4x^3 + 10x^2 + 12x + 9}\right) = ?$

- a) $\frac{x + 3}{-(x^3 + x^2 + x - 3)}$ b) $\frac{1}{1 - x^2}$
 c) $\frac{-1}{(x - 1)^2}$ d) $\frac{x + 3}{1 - x^2}$
 e) NOTA

19. The seventh term of an arithmetic sequence is $12x - 6y$ and the twelfth term is $17x + 4y$. What is the third term?

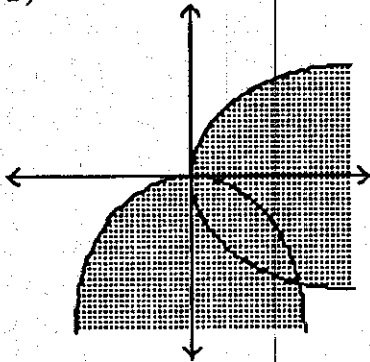
- a) $x + 2y$ b) $6x - 18y$ c) $8x - 14y$ d) $14x - 2y$ e) NOTA

20. If m and n are real numbers, under which of the following conditions will $\frac{m + ni}{1 + i}$ be a real number different from zero? (Note $i = \sqrt{-1}$)

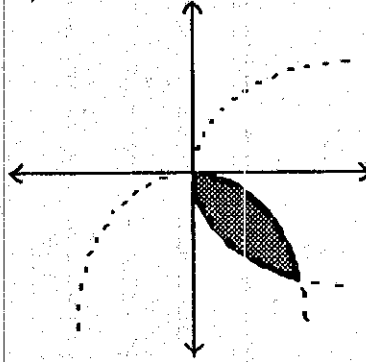
- a) $m = -n, m \neq 0$ b) $m \neq 0, n = 0$
 c) $m = n, m \neq 0$ d) $m = 2n, m \neq 0$
 e) NOTA

21. Which of the following could be the graph of the solution set for the system $\begin{cases} x \geq y^2 \\ y \leq -x^2 \end{cases}$?

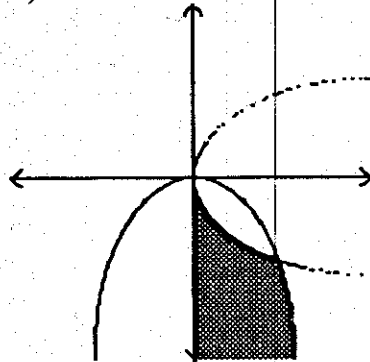
a)



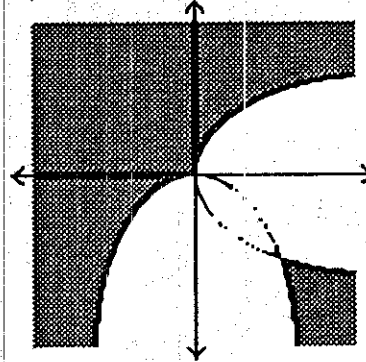
b)



c)



d)



e) NOTA

22. If three times the larger of two numbers is four times the smaller and the difference between the numbers is 8, then the larger of the two numbers is

- a) 16 b) 24 c) 32 d) 44 e) NOTA

23. Simplify: $\frac{2x - 2(x + 1)}{4(2(x+2))}$

- a) $-\frac{1}{16}$ b) $-\frac{1}{8}$ c) $\frac{1}{2}$ d) $\frac{1}{8}$ e) NOTA

24. Given $\log 2 = .3010$ and $\log 3 = .4771$ find $\log \sqrt{\frac{480}{.27}}$

- a) -.3751 b) .1249 c) .6249 d) 1.6249 e) NOTA

25. Let $f(x) = [x - 1]$ where $[x]$ is the greatest integer value of x . $g(x) = \sqrt{x - 2}$
 $h(x) = x^2 - 5$. Find $f(h(g(3.5)))$.

- a) -5 b) -4 c) 1 d) 5 e) NOTA

26. A parabola has directrix $y = 2$ and vertex at $(9,0)$. Find its equation.

a) $(x - 9)^2 = -8y$

b) $x^2 = -4(y - 9)$

c) $(x - 9)^2 = -4y$

d) $(x - 9)^2 = 8y$

e) NOTA

27. The set of all real x such that $\sqrt{x^2} = -x$ consists of

a) \emptyset

b) non positive reals

c) all Reals

d) $\{0\}$

e) NOTA

28. The range of the relation $\frac{(x + 3)^2}{4} + (y - 1)^2 = 1$ is

a) $-5 \leq y \leq -1$

b) $-3 \leq y \leq 0$

c) $0 \leq y \leq 1$

d) $0 \leq y \leq 2$

e) NOTA

29. Solve for x :
$$\begin{vmatrix} 2 & -1 & 1 \\ 3 & x & x \\ x & 2 & 5 \end{vmatrix} = 1$$

a) $\{5\}$

b) $\{-2, -5\}$

c) $\left\{\frac{3 \pm \sqrt{53}}{2}\right\}$

d) $\{-2, 5\}$

e) NOTA

30. Solve over \mathcal{R} : $\frac{1}{1 - x} = 1 - \frac{x}{x - 1}$

a) $x > 1$

b) $x = 2$

c) \mathcal{R}

d) \emptyset

e) NOTA