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Kaveh Hassani
Hillsborough High
Tampa, FL

Pre-Calculus Individual
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Tampa, Florida

- 1) If $f(x) + 2f\left(\frac{3-x}{x+1}\right) = x$, then find $f(0)$.
- a) -1 b) 1 c) 2 d) -2 e) NOTA
- 2) Which of the following is an even function?
- a) $\sin(90^\circ - x)$ b) $\tan x$ c) $\cos(90^\circ - x)$
d) $\sin(2x)$ e) NOTA
- 3) Solve for $x + y$, where $x, y \neq 0$ given the following:
- $$\begin{aligned} 3x + 2y &= 18xy \\ 5x + 3y &= 29xy \end{aligned}$$
- a) 7 b) $7/12$ c) $13/87$ d) $377/10$ e) NOTA
- 4) Solve for x to the nearest hundredth degree given:
- $$3 \cos x + 4 \sin x = 2 \quad 0^\circ \leq x < 360^\circ$$
- a) $\{13.29^\circ\}$ b) $\{119.55^\circ\}$ c) $\{13.29^\circ, 240.45^\circ\}$
d) $\{119.55^\circ, 346.71^\circ\}$ e) NOTA
- 5) If $\sin(2x) = A$ and $0 \leq x < 2\pi$, $-1 \leq A \leq 1$, and A is a constant, then there exists how many solutions for x ?
- a) 0 b) 1 c) 2 d) 4 e) NOTA
- 6) Give the best name for the graph formed by the following equation:
- $$2x^2 - xy - y^2 + 13x - y = -20$$
- a) hyperbola b) ellipse c) parabola
d) circle e) NOTA

7) Give the best name for the graph of the following function:

$$r = \frac{73}{(4 + 5\cos \theta)}$$

- a) limaçon b) hyperbolic spiral c) hyperbola
d) parabola e) NOTA

8) Find the angle (to the nearest hundredth degree) between vectors $(2,4,1)$ and $(1,4,3)$.

- a) 153.99° b) 169.84° c) 26.01° d) 10.16° e) NOTA

9) If $f(x) = x^3 + 6x + 5$ find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

- a) $3x^4 + 6x^2$ b) $3x^2 + 6$ c) $\frac{1}{4}x^4 + 3x^2 + 5x$
d) Doesn't exist e) NOTA

10) Find the equation of the locus of points equidistant from the line $y = .75x + 1$ and the point $(1,2)$.

- a) $6xy - 14x - 23y + 31 = 0$ b) $x^2 - 2x + 2y + 4 = 0$
c) $16x^2 + 24xy + 9y^2 - 56x - 92y + 124 = 0$ d) $2x + y^2 - 2y + 4 = 0$ e) NOTA

11) Which of the following is equivalent to the Boolean expression $(p' \vee q)' \rightarrow (q' \leftrightarrow r)'$.

- a) $(p \wedge q)' \rightarrow (q \leftrightarrow r)$ b) $(p' \wedge q) \rightarrow (q' \leftrightarrow r')$ c) $(p \vee q)' \rightarrow (q \leftrightarrow r')$
d) $(q' \leftrightarrow r') \rightarrow (p' \vee q)$ e) NOTA

12) Find all asymptotes for the graph $y = \frac{x^3 + 6x^2 + 7x + 9}{x^2 - 7x - 8}$

- a) $x=8, x=-1, y=x$ b) $x = 8, x = -1, y = x + 13$ c) $y=-1.125$
d) $x = 8, x = -1, y = -1.125$ e) NOTA

13) John, George, and Paul are playing a game of cards with the preceding order. John has a probability of winning of $\frac{1}{5}$ each turn, George of $\frac{1}{4}$, and Paul of $\frac{2}{9}$. If they play until someone wins, then what is the probability that John will win the game?

- a) $\frac{1}{5}$ b) $\frac{3}{8}$ c) $\frac{7}{75}$
 d) $\frac{3}{32}$ e) NOTA

14) Solve for x, given the following:

$$\begin{vmatrix} 4 & x & -5 \\ x-2 & -1 & -2 \\ -6 & 1 & x-6 \end{vmatrix} = 90$$

- a) $\{-1, 3\}$ b) $\{3, 6\}$ c) $\{0, 2, 6\}$ d) $\{-1, 3, 6\}$ e) NOTA

15) If $f(3x+1) = 9x^2 + 3x - 6$, then find the roots of $f(2x-1)$.

- a) $\{-2, 3\}$ b) $\{-.5, 2\}$ c) $\{-1, \frac{2}{3}\}$
 d) $\{-3, \frac{1}{3}\}$ e) NOTA

16) The hands of a clock meet at a certain angle at 7:23. When will they next meet at this angle?

- a) $7:28\frac{5}{11}$ b) $7:45\frac{2}{13}$ c) $8:28\frac{5}{11}$
 d) $8:45\frac{2}{13}$ e) NOTA

17) Evaluate the following ($[x]$ is equal to the greatest integer less than or equal to x):

$$[.999\dots] + [-\pi] + [e]$$

- a) -2 b) -1 c) 0 d) 1 e) NOTA

18) Find the area of the circle defined by $r = 3 \cos \theta$

- a) 2.25π b) 1.5 c) 3π
 d) 1.5π e) NOTA

19) Find the sum of all the coefficients in the expansion of $(4x-2y)^9$.

- a) 12 b) 256 c) 511
 d) 512 e) NOTA

20) The area of a parallelogram is 36 sq. ft. and the sides measure 58 ft. and 48 in. Find the measure of the obtuse angle between the sides to the nearest thousandth of a degree.

- a) 1.492 b) 18.020 c) 161.920
d) 178.518 e) NOTA

21) Two ships set out from the same port. One sets out at 1 PM, going 10 km/h north; the other leaves at 2:30 PM, going 15 km/h east. How far apart will they be at 4 PM? Give answer to nearest hundredth of a km.

- a) 37.5 b) 27.04 c) 54.08
d) 52.5 e) NOTA

22) Sally invests \$5,000 in two accounts. One has an interest rate of 6% and is compounded continuously, while the other has an interest rate of 8% and is compounded quarterly. How much (to the nearest cent) should she invest at 8%, if she expects to earn \$400 from interest in the first year?

- a) \$590.46 b) \$1837.37 c) \$3162.63
d) \$4409.54 e) NOTA

23) Ben and Greg can finish a scrapbook in 6 hours working together, Greg and Neal can finish one in 5 hours, and Neal and Ben can finish one in 5 1/2 hours. How long, in hours, will it take them to finish a scrapbook if they all work together at the previous rates?

- a) 1 149/181 b) 7 53/181 c) 3 117/181
d) 181/660 e) NOTA

24) What is the center of the circle that passes through points (6,4), (5,5), and (2,6)?

- a) (2, 1) b) (-2/3, -13/3) c) (11/2, 9/2)
d) (4, 5) e) NOTA

25) Which of the following is equal to: $\frac{\cos^4 x - \sin^4 x}{\cos^3 x - \sin^3 x}$ if $\cos x \neq \sin x$.

- a) $\sec x + \csc x$ b) $\cos x + \sin x$ c) $\frac{\cos x + \sin x}{1 + \cos x \sin x}$
d) $\cot x$ e) NOTA

26) What is the remainder when 7^{99} is divided by 13.

- a) 5 b) 9 c) 10 d) 12 e) NOTA

27) The second term of a geometric series is 16 more than the first. The third term is 80 more than the second. Find the second term.

- a) 4 b) 5 c) 20 d) 80 e) NOTA

28) Find: $\sin(\arcsin 12/13 + \arctan -3/5)$

- a) $\frac{\sqrt{1489}}{65}$ b) $\frac{12}{13} - \frac{3\sqrt{34}}{34}$ c) $\frac{\sqrt{705}}{65}$ d) $\frac{45\sqrt{34}}{442}$ e) NOTA

29) For which value of k listed below will $f(x) = 3kx^2 - 2k^2x + 9$ have more than one root?

- a) 0 b) $-1.5 - 1.5i\sqrt{3}$ c) $-1.5 + 1.5i\sqrt{3}$ d) $1.5 - 1.5i\sqrt{3}$ e) NOTA

30) Solve for x given the following: $4\sin^2x - 3\sin x + \cos 2x = 0$ and $0 \leq x < 2\pi$.

- a) $\{\pi/2, 3\pi/2\}$ b) $\{\pi/2\}$ c) $\{\pi/6, 5\pi/6\}$
d) $\{\pi/2, \pi/6, 5\pi/6\}$ e) NOTA