

Calculus exam

1.  $e < x < f$  (also accept  $(e, f)$  or  $[e, f]$  or  $e \leq x \leq f$ )

2.  $\frac{1}{b-a} \int_a^b \text{acceleration} \cdot dt = \frac{1}{\frac{\pi}{3} - \frac{\pi}{4}} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \text{accel} = \frac{12}{\pi} (\text{velocity}) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$   
 $\frac{12}{\pi} (\sec^2 \frac{\pi}{3} - \sec^2 \frac{\pi}{4}) = \frac{12}{\pi} (4 - 2) = \boxed{\frac{24}{\pi}}$

3.  $\frac{dp}{dt} = kp$  at  $t > 0$

Let 200 be  $t=0$   
 and 400 be  $t=2$   
 find  $p$  at  $t=5$

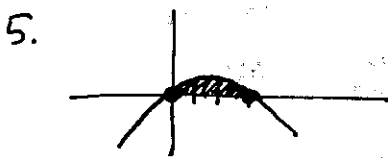
$$\left. \begin{aligned} \frac{1}{p} dp &= k dt \\ \ln p &= kt + c \\ \ln 200 &= 0 + c \end{aligned} \right\}$$

$$\begin{aligned} \ln p &= kt + \ln 200 \\ \ln 400 &= 2k + \ln 200 \\ \ln 2 &= 2k \\ \ln \sqrt{2} &= k \end{aligned}$$

$$\begin{aligned} \ln p &= (\ln \sqrt{2})t + \ln 200 \\ \ln p &= (\ln \sqrt{2})5 + \ln 200 \end{aligned}$$

$p = 1131.37$   
 $= \boxed{1131}$

4.  $\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x + c$   
 $f(x) = \boxed{x^2}$



5.  $\int_0^3 (3x - x^2) dx = 4.5$

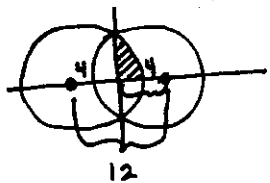
width = 3

$3x = 4.5$

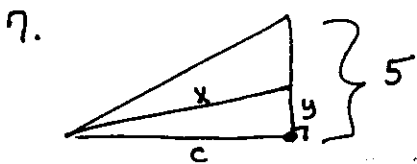
$x = 1.5 = \text{height}$

Perimeter =  $3(2) + 1.5(2) = \boxed{9}$

6.  $4 \int_0^2 (\text{left circle}) dx = 4 \int_0^2 \sqrt{64 - (x+6)^2} dx$   
 $= \boxed{29.012}$  rounded



Suggestion: Find equation of left circle and enter as  $y = \sqrt{64 - (x+6)^2}$  and then use integral capabilities of your calculator.



$BC = 2(CD)$

$BC + CD = 5$

$3CD = 5$

$CD = 5/3, BC = 10/3$

Answer =  $\frac{5\sqrt{349}}{698}$

$0 + 2y \frac{dy}{dt} = 2x \frac{dx}{dt}$

$2 \cdot \frac{5}{3} \cdot \frac{dy}{dt} = 2 \cdot \frac{\sqrt{349}}{3} \cdot \frac{dx}{dt}$

$-\frac{5}{2} = \sqrt{349} \cdot \frac{dx}{dt}$

$x^2 = 6 + \frac{25}{9}$

$x = \frac{\sqrt{349}}{3}$

$\approx \boxed{.134}$

Exam calculus

8. Arc =  $\int_1^4 \sqrt{1 - \frac{4}{9x^3}} dx = \int_1^4 \sqrt{1 - \frac{4}{3x^3}} dx$

Answer  $\approx 2.57598$

Yield =  $\sqrt{1+49}$

Answer =  $\boxed{10.142}$

9.  $x^2 + y^2 = 9(\cos^2 t + \sin^2 t)$  Square & total

$x^2 + y^2 = 9$

$\frac{dy}{dx} = -\frac{x}{y}$

10.  $y^2 dy = (x + 3x^2) dx$

$\frac{y^3}{3} = \frac{x^2}{2} + x^3 + c$

at  $y=6$  }  $72 = c$   
 $x=0$

$\frac{y^3}{3} = \frac{x^2}{2} + x^3 + 72$

$y^3 = \frac{3}{2}x^2 + 3x^3 + 216$

$y = \sqrt[3]{\frac{3}{2}x^2 + 3x^3 + 216}$

11.  $h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(5) \cdot (-3)$

$= 3(-3) = \boxed{-9}$

12.  $4x-1 = A(x-2)^2 + B(x)(x-2) + C(x)$

$4x-1 = Ax^2 - 4Ax + 4A + Bx^2 - 2Bx + Cx$

$A+B=0$

$-4A - 2B + C = 4$

$4A = -1$

$A = -\frac{1}{4}$

$\int_1^0 -\frac{1}{4} dx = -\frac{1}{4}x \Big|_1^0 = 0 + \frac{1}{4} = \boxed{\frac{1}{4}}$

13.  $\int_1^x (\ln x - 3) dx =$   
 $x \ln x - x - 3x \Big|_1^x$   
 $x \ln x - x - 3x - (1 \ln 1 - 1 - 3)$   
 $x \ln x - 4x + 4 = 4$   
 $x \ln x - 4x = 0$   
 $x(\ln x - 4) = 0$   
 $x=0$  or  $x = \boxed{e^4}$   
not in domain

14.  $\lim_{x \rightarrow \frac{\pi}{2}^-} (\tan x)^{\cos x} = k$

$\ln y = \cos x \cdot \ln \tan x =$   
 $\frac{\ln \tan x}{\sec x}$

$\lim_{x \rightarrow \frac{\pi}{2}^-} \ln y = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\ln \tan x}{\sec x}$

$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sec^2 x}{\tan x}$   
 $\sec x \cdot \tan x$

$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sec x}{\tan x} = 0$

So  $\lim_{x \rightarrow \frac{\pi}{2}^-} y = e^0 = 1$

$k=1$  Answer =  $\boxed{1}$

15.  $y = \cos x - x$

$y' = -\sin x - 1$   
 $y'' = -\cos x$   
 $y''(0.5) = -\cos(0.5) = -0.87758$

$= -0.87758$

$= \boxed{-0.87758}$