6 Calculus — Hills '96

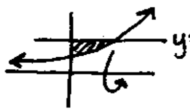
1. $\boxed{B}$

2. $f' = nx^{n-1} = nx^{-m}$ since $m = 1-n$ and $nx^{-m} = (1-m)x^{-m} = \boxed{D}$

3. $f(-1) = 6$; $f' = 4x-3$
$f(4) = 21$ so... $\frac{21-6}{4--1} = 4x-3$
$\frac{15}{5} = 4x-3$
$3/2 = x$ $\boxed{C}$

4. $\boxed{C}$ They are not differentiable over $[-1,1]$

5. $\boxed{E}$

6. 

$\pi\int r^2 dr = \pi\int_0^{\ln 4} (4^2 - e^{x \cdot 2})dx = \boxed{D}$

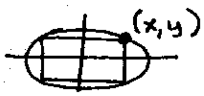
7. $\boxed{A}$

8. Area $= \pi ab$. $\frac{x^2}{4} + \frac{y^2}{25} = 1$. $\pi ab = \pi \cdot 2 \cdot 5$ $\boxed{B}$

9. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$. $f' = 2e^{2x}$
$\boxed{C}$: $4\int e^{2x}dx = 4 \cdot \frac{e^{2x}}{2} + c = 2e^{2x} + c$

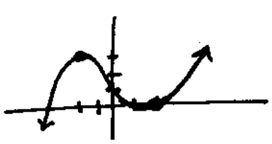
10. Perpendicular to the line $y=4$ at $(2,4)$ is the line $x=2$ $\boxed{C}$

11. $\frac{1}{b-a}\int_a^b \text{velocity} \cdot dt = \frac{1}{3-1}(\text{position})\Big|_1^3 = 1.8424 = \boxed{B}$

12. $\frac{x^2}{25} + \frac{y^2}{9} = 1$


Area of rectangle $= (2x)(2y) = 4x\left(\sqrt{9 - \frac{9}{25}x^2}\right)$
on calculator, the derivative's roots are at 3.5355338 (approx) or $2.5\sqrt{2}$
So rectangle is $5\sqrt{2}$ by $3\sqrt{2}$ $\boxed{B}$

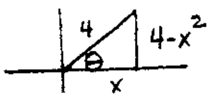
13. $f^{-1}(x)$: $y^2 - 3y + 2 = x$
$(2y-3)\,dy/dx = 1$
$\frac{dy}{dx} = \frac{1}{2y-3}$
when $x=2$ $y^2-3y+2=2$
$y=0$ or $y=3$
if $x \geq 3/2$ then for $f^{-1}(x)$, $y \geq 3/2$
so $\frac{dy}{dx} = \frac{1}{2(3)-3} = \frac{1}{3} = \boxed{B}$

14. 
Checking D: $\left.\begin{array}{l} f(-2) = 3 \\ f(2) = 0 \end{array}\right\}$ $\frac{3-0}{2--2} = \frac{3}{4}$. $\boxed{D}$ is incorrect.

15. $\frac{d^2y}{dx^2} = \cos x + e^{-x}$
$\frac{d^3y}{dx^3} = -\sin x - e^{-x} = \boxed{A}$

16. At $t=0$, $p = \$12{,}000$. $\frac{dp}{dt} = 100 + 300t^{\frac{1}{2}}$
$p = 100t + \frac{2}{3} \cdot 300t^{3/2} + c$.
$12000 = 100t + 200t^{3/2} + c$ at $t=0$
$p = 100t + 200t^{3/2} + 12000$
At $t=3$ $p = 13{,}339.23 = \boxed{C}$

17. $\frac{\Delta}{\text{sector}} = \frac{\frac{1}{2} \cdot 1 \cdot \sin\theta}{\frac{\theta}{2\pi} \cdot (\pi 1^2)} = \frac{\sin\theta}{\theta}$
As $\theta \to 0$ limit is 1 $\boxed{D}$

18.  $\int \frac{1}{x} \cdot \tan\theta \, dx = \int \frac{1}{x} \tan\theta d = \int \frac{1}{4} \sec\theta \tan\theta (-4\sin\theta) d\theta = -1 \int \tan^2\theta d\theta$

$= \int (1 - \sec^2\theta) d\theta = \theta - \tan\theta + c = \boxed{\cancel{A}}$

$\frac{4}{x} = \sec\theta$ so $\frac{1}{x} = \frac{1}{4}\sec\theta$

$\frac{x}{4} = \cos\theta$ so $-\sin\theta d\theta = \frac{1}{4} dx$

$-4\sin\theta d\theta = dx$

$\boxed{E}$ Hypotenuse wrong in solution

19. Let $u = \frac{1}{x} = x^{-1}$, $du = -\frac{1}{x^2} dx$ so $6\int \frac{e^{\frac{1}{x}}}{x^2} dx = -6\int e^u du$ $\boxed{D}$

20. $|f(x)|$ reflects up to positive y-coordinate. $f(|x|)$ will maintain a negative y-coordinate $\boxed{C}$. Since $\int_3^5 f(x)dx = -2$ then $\int_3^5 f(|x|) = 2$

21. $3\sum_{i=1}^{n} i - \sum_{i=1}^{n} 2 = 3\left(\frac{n}{2}(n+1)\right) - 2n = \frac{3n^2 - n}{2} = \boxed{D}$

22. $y_2 = y_1 - \frac{f(y_1)}{f'(y_1)}$ when $y_1 = 1^{st}$ approx. So $y_2 = r_1 - \frac{\tan(r_1)}{\sec^2(r_1)} = r_1 - \sin r_1 \cos r_1 =$

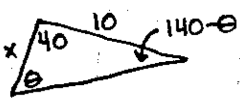
$r_1 - \frac{1}{2}\sin(2r_1) = \boxed{B}$

23. Tan line: $y - 3 = (2x-2)(x-3)$; $y = 2x^2 - 8x + 9$ → $x^2 - 6x + 8 = 0$ → $x = 2$ or $x = 4$

Curve: $y = x^2 - 2x + 1$; $x^2 - 2x + 1 = 2x^2 - 8x + 9$ → $(x-4)(x-2) = 0$

Slope $= 2x - 2 = 2$ or 6 Sum $= 8$ $\boxed{D}$

Alternate method: Set $2x - 2 = \frac{\Delta y}{\Delta x} = \frac{(x-1)^2 - 3}{x - 3}$

24.  $\frac{\sin(140-\theta)}{x} = \frac{\sin\theta}{10}$

$x = \frac{10(\sin(140-\theta))}{\sin\theta}$

at 5 minutes, $\theta = 35°$, $x = 16.8404$

$\frac{dx}{dt} = \frac{10[\sin\theta\cos(140-\theta)(-1d\theta) - \cos\theta\sin(140-\theta)d\theta]}{\sin^2\theta}$

$dx = .03677956 \rightarrow .037$

(Remember that $d\theta = \pi/180$

25. (ii) $y' = 2x$ is monotonic increasing

(iii) $y' = \frac{x-1}{\sqrt{x^2-2x+5}}$ is monotonic increasing

$\boxed{C}$

26. $f(x) = e^x$ so $\int \ln(e^x) dx = \int x dx = \boxed{D}$

27. $(\sin\theta + \cos\theta)^2 = .16$

$1 + 2\sin\theta\cos\theta = .16$

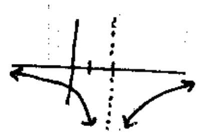
$\sin 2\theta = -.84$ → $\int_0^5 (-.84)dx = \boxed{B}$

28. $k = \sqrt{42 - k}$

$k^2 + k - 42 = 0$

$k = -6$ or $k = 7$ → The graph of $7x^2 + 14x = f(x)$ has a rel. min. at $x = -1$

$\boxed{B}$

29. The derivative is $\frac{-2}{(2x-4)^2}$  $\boxed{C}$

30. $\int \sin^2 x dx = \int \left(\sqrt{\frac{1 - \cos 2x}{2}}\right)^2 dx$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C_1$

Step 2: $\int \left(\frac{1}{2}x - \frac{1}{4}\sin 2x + C_1\right) dx$

$= \frac{1}{4}x^2 + \frac{1}{8}\cos 2x + C_1 x + C_2$

$\boxed{C}$