Multiplying a complex number z by $e^{i\theta}$ rotates z counter clockwise about the origin θ radians. Using this information find the following:

Let $A=$ The magnitude of the new point when $1+2i$ is rotated $\frac{\pi}{3}$ radians counterclockwise about the origin

Let $B=$ $\left|\frac{(1+i)^{16}}{(1-i)^{16}}\right|$

Let $C=$ The argument of $\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{6}-\sqrt{2}}{4}i$ for the interval $(0,\pi/2)$

Let $D=$ $(3+4i)e^{\pi i/2}$

$\frac{AB}{C|D|}$ can be expressed as $\frac{x}{\pi\sqrt{y}}$ where x and y are relatively prime natural numbers. Find $x+y$.

Precalculus Team Question 1

Multiplying a complex number z by $e^{i\theta}$ rotates z counter clockwise about the origin θ radians. Using this information find the following:

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$\frac{AB}{C|D|}$ can be expressed as $\frac{x}{\pi\sqrt{y}}$ where x and y are relatively prime natural numbers. Find $x+y$.

Use $f(x) = \dfrac{x^3 - 11x^2 + 6x + 4}{x^3 - 1}$ to answer the following:

Let $A=$ The total number of asymptotes of f

Let $B=$ $\lim_{x \to 2} f(x)$

Let $C=$ 0 if $f(x)$ is even and 1 otherwise

Let $D=$ The sum of the solutions to $\alpha x^2 + \beta x + C = 0$ if $y = \alpha$ is the horizontal asymptote and $\beta =$ the number of removable discontinuities of $f(x)$, and C is the value of C given above.

Find the value of the determinant $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Precalculus Team Question 2

Use $f(x) = \dfrac{x^3 - 11x^2 + 6x + 4}{x^3 - 1}$ to answer the following:

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Let $D=$ The sum of the solutions to $\alpha x^2 + \beta x + C = 0$ if $y = \alpha$ is the horizontal asymptote and $\beta =$ the number of removable discontinuities of $f(x)$, and C is the value of C given above.

Find the value of the determinant $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Let $A=$ The area of a triangle with two sides being 3 and the included angle is $\frac{\pi}{6}$

Let $B=$ The third side of an isosceles triangle with congruent sides of $\sqrt{3}$ and the angle between the congruent sides is $\frac{2\pi}{3}$

Let $C=$ The area of a regular hexagon with a side length 3

Let $D=$ $\left(\sin\frac{\pi}{8}+\cos\frac{\pi}{8}\right)^2$

Find the value of $4A-3B+C\sec\left(\frac{\pi}{6}\right)+(D-1)\csc\left(\frac{\pi}{4}\right)$

Precalculus Team Question 3

Let $A=$ The area of a triangle with two sides being 3 and the included angle is $\frac{\pi}{6}$

Let $B=$ The third side of an isosceles triangle with congruent sides of $\sqrt{3}$ and the angle between the congruent sides is $\frac{2\pi}{3}$

Let $C=$ The area of a regular hexagon with a side length 3

Let $D=$ $\left(\sin\frac{\pi}{8}+\cos\frac{\pi}{8}\right)^2$

Find the value of $4A-3B+C\sec\left(\frac{\pi}{6}\right)+(D-1)\csc\left(\frac{\pi}{4}\right)$

$$A(x) = \sin\big(\tan^{-1} x\big)$$
$$B(x) = \cos\big(\sin^{-1} x\big)$$
$$C(x) = \sin\big(\cos^{-1}(\cos\big(\sin^{-1} x\big))\big)$$
$$D(x) = \sin^{-1}(\sin x)$$

Assume the traditional ranges for the inverse trig functions. Let P be the maximum value of all the functions evaluated at $x = 1$ and let R be the minimum value of the functions evaluated at $x = -1$, find the value of $P - R$.

Precalculus Team Question 4

$$A(x) = \sin\big(\tan^{-1} x\big)$$
$$B(x) = \cos\big(\sin^{-1} x\big)$$
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Assume the traditional ranges for the inverse trig functions. Let P be the maximum value of all the functions evaluated at $x = 1$ and let R be the minimum value of the functions evaluated at $x = -1$, find the value of $P - R$.

Let $f(x) = \sin^{-1}(x^2 - 1)$
The domain of $f(x)$ can be written as $[-A, A]$, find A.
The range of $f(x)$ is $[-B, B]$, find B
Let C be the maximum value of $|f(x)|$ on its domain
Let D be the maximum value of the function $g(x) = \sqrt{\sin x + 1}$
Find the value of $\sin^{-1}(A/D) + \cos(B) + \sin(C)$

Precalculus Team Question 5

Let $f(x) = \sin^{-1}(x^2 - 1)$
The domain of $f(x)$ can be written as $[-A, A]$, find A.
The range of $f(x)$ is $[-B, B]$, find B
Let C be the maximum value of $|f(x)|$ on its domain
Let D be the maximum value of the function $g(x) = \sqrt{\sin x + 1}$
Find the value of $\sin^{-1}(A/D) + \cos(B) + \sin(C)$

The hyperbola $4x^2 - 9y^2 - 8x + 54y - 113 = 0$ is rotated 135° counterclockwise about the origin.
Let A be length of the latus rectum
Let B be the distance between the new location of the vertices
Let C be the sum of the x–coordinates of the foci on the rotated graph
Let D be the sum of the y–coordinates of the foci on the rotated graph
Find the value of $3A - B + C - 2D$

Precalculus Team Question 6

The hyperbola $4x^2 - 9y^2 - 8x + 54y - 113 = 0$ is rotated 135° counterclockwise about the origin.
Let A be length of the latus rectum
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Let C be the sum of the x–coordinates of the foci on the rotated graph
Let D be the sum of the y–coordinates of the foci on the rotated graph
Find the value of $3A - B + C - 2D$

Let $0 < x < \pi/2$, simplify each expression to a single trigonometric function.

$A = \dfrac{\sin 2x}{2(1-\sin^2 x)}$

$B = \dfrac{\sqrt{2-2\cos(2x)}}{\sin(2x)}$

$C = \dfrac{1}{\sqrt{2}}\left[\sin\left(x+\dfrac{\pi}{4}\right)+\cos\left(x+\dfrac{9\pi}{4}\right)\right]$

$D = \dfrac{z-\bar{z}}{2}$ where $z = \cos x + i\sin x$ and $\bar{z}$ denotes the complex conjugate of z.

Find the value of $A^2 - B^2 + C^2 - D^2$

Precalculus Team Question 7

Let $0 < x < \pi/2$, simplify each expression to a single trigonometric function.

$A = \dfrac{\sin 2x}{2(1-\sin^2 x)}$

$B = \dfrac{\sqrt{2-2\cos(2x)}}{\sin(2x)}$

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$D = \dfrac{z-\bar{z}}{2}$ where $z = \cos x + i\sin x$ and $\bar{z}$ denotes the complex conjugate of z.

Find the value of $A^2 - B^2 + C^2 - D^2$

A parabola passes through the points $(1,3),(2,5),(3,11)$ and has either vertical or horizontal symmetry.

Let $A =$ The number of possible parabolas

Let $B =$ The distance between the focus and directrix of the parabola with vertical symmetry; if one does not exist, let $B = 0$.

Let $C =$ The sum of (i) the distances from the point $(2,5)$ to the point $\left(1, \frac{25}{8}\right)$ and (ii) the shortest distance from $(2,5)$ to the line $y = \frac{23}{8}$

Find the value of $3A + 8B + 4C$

Precalculus Team Question 8

A parabola passes through the points $(1,3),(2,5),(3,11)$ and has either vertical or horizontal symmetry.

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Find the value of $3A + 8B + 4C$

A radioactive isotope of Erictonium-266 has a half life of 1 nanosecond. A lab has 1 gram of Erictonium at time $t = 0$. Let T be the amount of time (in nanoseconds) after $t = 0$ when there is $1/e$ grams of Erictonium left.

Let $A = \sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right)$

Let $O = p\left(\frac{1}{\ln 5}\ln\left(\sum_{n=0}^{\infty}\frac{1}{n!}\right)\right)$ if $p(x) = 5^x$

Let $f(x) = \frac{(1+\cot^2 x)(1-\cos 2x)}{3-\cos 2x}$ and let $N = \lim_{x\to 0} f(x)$

Find the value of $NOTA$

Precalculus Team Question 9

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Let $f(x) = \frac{(1+\cot^2 x)(1-\cos 2x)}{3-\cos 2x}$ and let $N = \lim_{x\to 0} f(x)$

Find the value of $NOTA$

Suppose $\mathbf{M} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, let $A = \left|\mathbf{M}^{-1}\right|$ when $\theta = \frac{\pi}{4}$

If vector $\vec{\mathbf{u}} = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$ and vector $\vec{\mathbf{v}} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$, let B be the radian measure of the acute angle between $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

$\ell(x) = \dfrac{\sin x - \cos x}{1 + \sin 2x}$, if $u = \sin x - \cos x$, let $r(u)$ be $\ell(x)$ rewritten in terms of u, let $C = r(1)$

If $t = \sec x + \tan x$, let $p(t)$ denote the value of $\cos x$ expressed as a function of t. Let $D = p(1)$ (Hint: Look at $1/t$)

Find the value of $\cos(3ABCD)$

Precalculus Team Question 10

Suppose $\mathbf{M} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, let $A = \left|\mathbf{M}^{-1}\right|$ when $\theta = \frac{\pi}{4}$

If vector $\vec{\mathbf{u}} = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$ and vector $\vec{\mathbf{v}} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$, let B be the radian measure of the acute angle between $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

$\ell(x) = \dfrac{\sin x - \cos x}{1 + \sin 2x}$, if $u = \sin x - \cos x$, let $r(u)$ be $\ell(x)$ rewritten in terms of u, let $C = r(1)$

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Find the value of $\cos(3ABCD)$

A Ferris wheel at its lowest point is 2 meters off the ground and its highest point is 28 meters off the ground, this Ferris wheel happens to be very fast and completes 10 revolutions every minute. Let $h(t)$ be the height of a point that starts at the bottom of the Ferris wheel at $t = 0$ after t minutes.

Let $A =$ The number of times the Ferris wheel goes around in a 10 minute time period

Let $B =$ $h(1)$

Let $C =$ The period in minutes

Let $D =$ How many minutes after $t = 0$ it takes to reach the maximum (for the first time)

Let $E =$ The time it takes to complete one revolution (in minutes)

Let $R =$ The radius of the Ferris wheel (in meters)

Find the value of $10ABCDER$

Precalculus Team Question 11

A Ferris wheel at its lowest point is 2 meters off the ground and its highest point is 28 meters off the ground, this Ferris wheel happens to be very fast and completes 10 revolutions every minute. Let $h(t)$ be the height of a point that starts at the bottom of the Ferris wheel at $t = 0$ after t minutes.

Let $A =$ The number of times the Ferris wheel goes around in a 10 minute time period

Let $B =$ $h(1)$

Let $C =$ The period in minutes

Let $D =$ How many minutes after $t = 0$ it takes to reach the maximum (for the first time)

Let $E =$ The time it takes to complete one revolution (in minutes)

Let $R =$ The radius of the Ferris wheel (in meters)

Find the value of $10ABCDER$

Assuming all has gone well we have now reached the end of the team round. It seems only fitting that we conclude with a joke. What noise does a drowning analytic number theorist make?

Let S be the set of all ordered pairs (x, y) such that $\log(x + y) = \log x + \log y$. Let L be the number of ordered pairs in S, if S is infinite, let $L = e$.

Consider the equation $e^{2x^2} - 2e^{x^2} - 3 = 0$, let O be product of the real solutions for x

In the equation $3^{-2x} + 3^{-x}(\pi - e) - \pi e = 0$, let G be the sum of the real solutions

Find the value of $LOGLOGLOG$ (coincidentally, this is also the answer to the joke)

Precalculus Team Question 12

Assuming all has gone well we have now reached the end of the team round. It seems only fitting that we conclude with a joke. What noise does a drowning analytic number theorist make?

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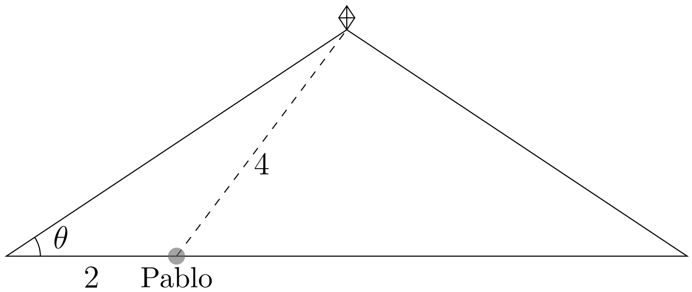
Find the value of $LOGLOGLOG$ (coincidentally, this is also the answer to the joke)

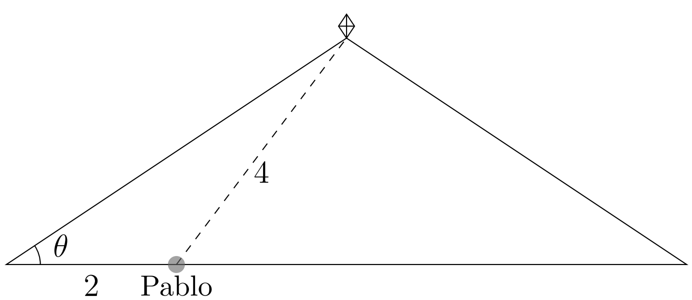
Pablo is flying a kite as shown in the image below (not drawn to scale)



In the diagram, the horizontal line represents the ground and $\theta = \tan^{-1}(1/2)$. If h is the height of the kite, find $\lfloor h \rfloor$. (Where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

Precalculus Team Question 13

Pablo is flying a kite as shown in the image below (not drawn to scale)



In the diagram, the horizontal line represents the ground and $\theta = \tan^{-1}(1/2)$. If h is the height of the kite, find $\lfloor h \rfloor$. (Where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

Consider the following statements for all real values of x.

A: $(\cos x + \sin x)^2 = 1 + \cos 2x$

B: $\dfrac{\sin x}{1+\cos x} = \tan\dfrac{x}{2}$

C: $\displaystyle\sum_{n=0}^{6} t^n = 0$ has no real solutions for t.
(Hint: Remember geometric series)

D: $(\cos x + \sin x)(\cot x + \tan x) = \cot 2x$

If the statement is true, let 1 be the value of the letter next to it, if the statement is false, let -1 be the value of the letter. Find the sum of the values of the letters.

Precalculus Team Question 14

Consider the following statements for all real values of x.

A: $(\cos x + \sin x)^2 = 1 + \cos 2x$

B: $\dfrac{\sin x}{1+\cos x} = \tan\dfrac{x}{2}$

C: $\displaystyle\sum_{n=0}^{6} t^n = 0$ has no real solutions for t.
(Hint: Remember geometric series)

D: $(\cos x + \sin x)(\cot x + \tan x) = \cot 2x$

If the statement is true, let 1 be the value of the letter next to it, if the statement is false, let -1 be the value of the letter. Find the sum of the values of the letters.

The Weierstrass substitution is a very powerful substitution, one can turn a trigonometric equation/expression into a simple polynomial. This substitution is made by letting $t = \tan\dfrac{x}{2}$ and putting the other trigonometric functions in terms of t

If $\sin(x) = f(t)$, $\cos x = g(t)$, $\tan x = h(t)$, find the value of $f(1) - g(1) + h(2)$.

Precalculus Team Question 15

The Weierstrass substitution is a very powerful substitution, one can turn a trigonometric equation/expression into a simple polynomial. This substitution is made by letting $t = \tan\dfrac{x}{2}$ and putting the other trigonometric functions in terms of t

If $\sin(x) = f(t)$, $\cos x = g(t)$, $\tan x = h(t)$, find the value of $f(1) - g(1) + h(2)$.