1. **D.**
2. **E.**
3. **B.**
4. **E.**
5. **C.**
6. **D.**
7. **D.**
8. **B.**
9. **C.**
10. **B.**
11. **C.**
12. **C.**
13. **C.**
14. **B.**
15. **D.**
16. **C.**
17. **D.**
18. **D.**
19. **A.**
20. **C.**
21. **C.**
22. **B.**
23. **C.**
24. **B.**
25. **C.**
26. **E.**
27. **D.**
28. **C.**
29. **A.**
30. **D.**

1. **D.** All we care about is the coefficient of x, combining the like terms we get $(1+\gamma)x$, note that since $\log(\gamma)$ is a real number $\gamma \neq -1$ so $1+\gamma \neq 0$. From this we have that the period is $\dfrac{2\pi}{1+\gamma}$

2. **E.** $P = \dfrac{2\pi}{2\pi/3} = 3, A = 2, B = -\dfrac{-\pi/12}{2\pi/3} = \dfrac{1}{8}, L = -4$. So $PABLA = 3 \cdot 2 \cdot \dfrac{1}{8} \cdot -4 \cdot 2 = -6$.

3. **B.** Let $u = e^x$ for easier calculation, we are now trying to find the values of u such that $\dfrac{u + \frac{1}{u}}{2} = 2$. This equation can be rewritten as $u^2 - 4u + 1 = 0$ which is just a simple quadratic so $u = \dfrac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$. From the substitution we get $x = \ln u$ so $x = \ln\left(2 \pm \sqrt{3}\right)$. At this point we should note that $2 \pm \sqrt{3} > 0$ so we can actually take the natural log. Thus the sum of the solutions is $\ln\left(2+\sqrt{3}\right) + \ln\left(2 - \sqrt{3}\right) = \ln 1 = 0$

4. **E.** First note that $\sin(\pi - x) = \sin\pi\cos x - \cos\pi\sin x = \sin x$

   I. Since $\sin(\pi - x) = \sin x$ we have $\ln(\sin(\pi - x)) = \ln(\sin x)$ so $\ln(\sin x)$ is symmetric on the interval from 0 to π

   II. $\ln\left(\cos\left(\dfrac{\pi}{2} - x\right)\right) = \ln(\sin x) \neq \ln(\cos x)$ so it is not symmetric on the interval.

   III. $\cos(\pi - x) = \cos\pi\cos x + \sin\pi\sin x = -\cos x$, so $\cos x$ is not symmetric on the interval.

   IV. From the cofunction identities we have $\sqrt{\tan\left(\dfrac{\pi}{2} - x\right)} + \sqrt{\cot\left(\dfrac{\pi}{2} - x\right)} = \sqrt{\cot x} + \sqrt{\tan x}$ so this is symmetric on the interval.

   V. Again from the cofunction identities this will be symmetric

   VI. $\sin^n(\pi - x)\cos^{2n}(\pi - x) = \sin^n x(-\cos x)^{2n}$, since n is a positive integer, $2n$ will always be even. From this we get that the function is symmetric

   So the only functions that are symmetric are I, IV, V, VI

5. **C.** Dividing 1989 by 360 we get that it goes in 5 times with a remainder of $189°$. The reference angle is the angle that is made with the x axis so the answer is $9°$.

6. **D.** Recall that a complex number ω can be expressed as $re^{i\theta}$ with r being the magnitude of ω and θ the argument of ω. Using this together with De'Moivre's theorem we can find what the magnitude of z is. First let $\omega = 1 + e^{\frac{\pi i}{3}}$. We then have

$$\omega = 1 + e^{\frac{\pi i}{3}} = \frac{3}{2} + \frac{\sqrt{3}}{2}i$$

   We want to put this in terms of $re^{i\theta}$ so we need to find r which is as follows:

$$r = |\omega| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$$

To find the argument θ we can either look at $\tan\theta = y/x$ or we can look at $\sin\theta$ or $\cos\theta$. The easiest way tends to look at $\tan\theta$ and then note what quadrant ω is in. Since ω has both a positive real part and imaginary part we know that ω lies in the first quadrant so $\theta \in \left[0, \frac{\pi}{2}\right]$. With this we have $\tan\theta = y/x = \frac{3/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$ so $\tan\theta = \frac{1}{\sqrt{3}}$ in the first quadrant and thus $\theta = \pi/6$. Hence $\omega = \sqrt{3}e^{\frac{\pi i}{6}}$. Since $z = \omega^5$ we have

$$z = \left(\sqrt{3}e^{\frac{\pi i}{6}}\right)^5 = 3^{5/2}e^{\frac{5\pi i}{6}}$$

Since the magnitude of $e^{\frac{5\pi i}{6}}$ is 1, we know $|z| = 3^{5/2} = 9\sqrt{3}$.

7. **D.** For a limit to exist, the limits from both the left and right must exist and be the same value. To the left of 0, say any negative number, $x/x = 1$. To the right of 0, say any positive number, $x/x = 1$. Thus the limits from the left and right of 0 are both 1 so $\lim\limits_{x\to 0} \frac{x}{x} = 1$.

8. **B.** Hopefully this will be a telescoping series (it is) so partial fractioning

$$\frac{1}{(n+2)(n+1)} = \frac{A}{n+2} + \frac{B}{n+1}$$

We find that $A(n+1) + B(n+2) = 1$ so $A + B = 0$ and $A + 2B = 1$. From this $A = -1$ and $B = 1$. The sum can then be rewritten as

$$\begin{aligned}\sum_{n=0}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) &= \sum_{n=0}^{\infty}\frac{1}{n+1} - \sum_{n=0}^{\infty}\frac{1}{n+2} \\ &= 1 + \cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \ldots - \left(\cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \ldots\right) \\ &= 1\end{aligned}$$

9. **C.** The easiest way to do this question is to use our knowledge of trigonometry, a bit of algebra, and guessing and checking. The one that eliminates the most is guessing and checking, let $x = 0$, in A, $\cot^{-1} 0 = \frac{\pi}{2}$ and $\frac{\pi}{2} + \frac{\pi}{2} \neq 0$ so we can eliminate that one. Similarly in B, $\sin^{-1} 1 = \frac{\pi}{2} \neq 0$. As for C, $2\tan^{-1} 1 - \frac{\pi}{2} = \frac{\pi}{2} - \frac{\pi}{2} = 0$ so we cannot eliminate it yet. Continuing with D, $\cos^{-1} 0 = \frac{\pi}{2} \neq 0$ so all that remains is C. Although this looks highly unusual we can keep guessing and checking values and find that it always works. For a more formal proof, let $\theta = 2\tan^{-1}\left(\sqrt{1+x^2} + x\right) - \frac{\pi}{2}$, we

proceed as follows:

$$\begin{aligned}\tan\theta &= \tan\left[2\tan^{-1}\left(\sqrt{1+x^2}+x\right)-\frac{\pi}{2}\right]\\ &= \frac{\sin\left[-\left(\frac{\pi}{2}-2\tan^{-1}\left(\sqrt{1+x^2}+x\right)\right)\right]}{\cos\left[-\left(\frac{\pi}{2}-2\tan^{-1}\left(\sqrt{1+x^2}+x\right)\right)\right]}\\ &= \frac{-\cos\left(2\tan^{-1}\left(\sqrt{1+x^2}+x\right)\right)}{\sin\left(2\tan^{-1}\left(\sqrt{1+x^2}+x\right)\right)}\\ &= -\cot\left(2\tan^{-1}\left(\sqrt{1+x^2}+x\right)\right)\\ &= -\frac{1-\left(\sqrt{1+x^2}+x\right)^2}{2\left(\sqrt{1+x^2}+x\right)}\\ &= -\frac{1}{2}\cdot\frac{1}{\sqrt{1+x^2}+x}+\frac{1}{2}\left(\sqrt{1+x^2}+x\right)\\ &= -\frac{1}{2}\left(\sqrt{1-x^2}-x\right)+\frac{1}{2}\left(\sqrt{1+x^2}+x\right)\\ &= x\end{aligned}$$

Thus $\tan\theta = x$ and therefore $\theta = \tan^{-1} x$.

10. **B.** The problem is that Da Stow multiplied on the right by $\mathbf{A^{-1}}$ when he should have multiplied on the left. So $\mathbf{x} = \mathbf{A^{-1}B}$. Although A gives the correct solution to the system, it is not correct since the question asked what Da Stow did wrong.

11. **C.** Recall that when you partial fraction $\frac{1}{(x+1)(x^2+1)}$ there is $\frac{Ax+B}{x^2+1}$ since the x^2+1 is irreducible, a similar concept applies for the t^4+1 in this question. We need to keep our system of equations "balanced," the same number of equations as variables. When we multiply by the denominator we get

$$-2t = A(t^4+1) + f(t)(t+1)$$

so the only logical choice for $f(t)$ is a cubic polynomial so that A will not be the only thing contributing to a quartic term. With this in mind the rest is algebra, multiplying

$$A(t^4+1) + (Bt^3 + Ct^2 + Dt + E)(t+1)$$

we get

$$At^4 + A + Bt^4 + (B+C)t^3 + (C+D)t^2 + (D+E)t + E.$$

Combining the like terms

$$(A+B)t^4 + (B+C)t^3 + (C+D)t^2 + (D+E)t + (A+E) = -2t.$$

So the following system arises

$$A + B = 0 \tag{1}$$
$$B + C = 0 \tag{2}$$
$$C + D = 0 \tag{3}$$
$$D + E = -2 \tag{4}$$
$$A + E = 0 \tag{5}$$

The best way to solve this system is to get 2 equations with the same 2 variables, once that is solved the rest come easily. Subtraction (1) and (2) we find $A - C = 0$ so $A = C$. Then doing (3) – (4) we get $C - E = 2$ and we rewrite (5) as $C + E = 0$. We then have a simple system with just C and E so $C = 1, E = -1$. Finding the rest of the variables we have

$$A = 1, B = -1, C = 1, D = -1, E = -1.$$

We then have $f(t) = -t^3 + t^2 - t - 1$ and $f(1) = -2$.

12. **C.** $\log_2 x + \log_2(x-3) = \log_2(x^2 - 3x)$, since this is equal to 2, $x^2 - 3x = 2^2$. Subtracting the 4 over and factoring we get $x = 4, -1$. Checking these values in the original equation we see that only 4 works so the sum of the solutions is just 4

13. **C.** By definition, $\mu\big(x; f(x)\big) = \sum_{x=0}^{\infty} x \cdot \frac{1}{2^x}$. To evaluate this sum we first consider the following derivation of the sum of an infinite geometric series with $|r| < 1$

$$\begin{aligned} S &= a(1 + r + r^2 + r^3 + \dots) \\ S/r &= a(\frac{1}{r} + 1 + r + r^2 + \dots) \\ S/r - S &= \frac{a}{r} \\ S &= \frac{a}{1-r} \end{aligned}$$

With this in mind we can do the question. Let S equal the desired quantity, we proceed as follows:

$$\begin{aligned} S &= \frac{1}{2} + \frac{2}{4} + \frac{3}{2^3} + \dots \\ S/2 &= \qquad \frac{1}{4} + \frac{2}{8} + \dots \\ S - S/2 &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \\ S/2 &= \frac{\frac{1}{2}}{1 - \frac{1}{2}} \\ S &= 2 \end{aligned}$$

14. **B.** By definition, $\mu\big(e^{tx}; g(x)\big) = \sum_{x=0}^{\infty} e^{tx} g(x) = \sum_{x=0}^{\infty} \frac{e^{tx}e^{-10}(10^x)}{x!}$. Recall that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, we can rewrite our expression as $e^{-10}\sum_{x=0}^{\infty} \frac{(10e^t)^x}{x!}$ and to make things more familiar we can change the letter to get $e^{-10}\sum_{n=0}^{\infty} \frac{(10e^t)^n}{n!}$. From this we see $M(t) = e^{-10}e^{10e^t} = e^{10(e^t-1)}$. Finally, $M(1) = e^{10(e-1)}$.

15. **D.** Since we are looking to show the statement for every nonnegative integer we start with the first nonnegative integer which is 0. So our base case would be showing that the statement is true for $n = 0$. We now note why the other answer choices are incorrect. If we show the statement is true at $n = 100$ then we don't know about if the statement holds $n = 0, \ldots, 99$ so even after we have performed the induction step, we will have only shown the statement for $n \geq 100$ and not all nonnegative integers. Since the statement only mentioned nonnegative integers, $n = -1$ is not even within the scope of our discussion. As for C, that is the entirety of the proof by induction and not the base case.

16. **C.** The standard way of proving induction is to prove the base case and then assume the statement is true at some fixed value of n and then show said statement is true at one more than said n. In other words, assume true at $n = m$ and prove it for $n = m+1$. We now discuss why the answer is C by showing that all the other answers are valid induction steps. To ease writing, let the statement be S.

    A. This is a valid induction step as it is the same thing as the traditional way. Assuming S is true at a fixed n which $n = m - 1$ then showing it is true for one more which is $n = m - 1 + 1 = m$.

    B. If we assume S is false at $n = m + 1$ and then show S is false at $n = m$ that is equivalent to showing that if S is false at one value of m then S is false at every preceding value. So if the statement is true at even one value, which is what the base case shows, then it follows that every following value will be true. Alternatively, in a conditional statement "if p then q" the contrapositive "if not q then not p" is equivalent to the original statement.

    D. This is the standard way of proving induction so it is clearly a valid induction step.

    All that remains is C which if C were actually performed then it would show that if S is false at some n then S is false at each of the following values of n. This does not show anything about the statement being true, the equivalent statement is that if S is true at $n = m+1$ then S is true at $n = m$. This shows that if S is true at some value of n then it is true at all preceding values. This does not show it is true for any nonnegative n.

17. **D.** Recall from the pythagorean identity, $\cos^2 t + \sin^2 t = 1$, using this fact we have $\cos t = x^{1/n}$ and $\sin t = y^{1/n}$ so $x^{2/n} + y^{2/n} = 1$. From this we see $\alpha = \frac{2}{n}$ and $\beta = 1$ so $\frac{\alpha}{\beta} = \frac{2}{n}$.

18. **D.** Due to the possible ranges for α and β it is valid to say $\alpha = 2\tan^{-1}\left(\frac{5}{4}\right)$ and $\beta = 2\tan^{-1}\left(\frac{3}{5}\right)$. There are a variety of ways to do this question:

Method 1. Putting the expression in terms of tangent of α or β over 2. We have that

$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = 2\cdot\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cdot\cos^2\frac{\alpha}{2}$$

which can be rewritten as

$$\frac{2\tan\frac{\alpha}{2}}{\sec^2\frac{\alpha}{2}} = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}.$$

Similarly we can find that

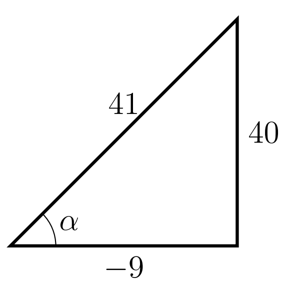
$$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1$$

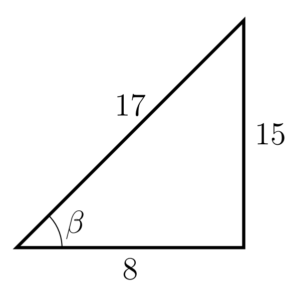
so

$$\frac{2}{\tan^2\frac{\alpha}{2}+1} - 1 = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}.$$

From this we find $\sin\alpha = \frac{2\cdot\frac{5}{4}}{1+\left(\frac{5}{4}\right)^2} = \frac{40}{41}$ and we can draw a triangle, note that $\cos\alpha < 0$ because it is in the second quadrant. Similarly we have $\cos\beta = \frac{1-\left(\frac{3}{5}\right)^2}{1+\left(\frac{3}{5}\right)^2} = \frac{25-9}{25+9} = \frac{8}{17}$.

We can draw the following triangles (not to scale)





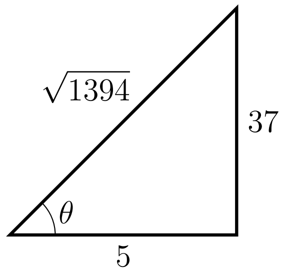
From here we obtain

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{40}{41}\cdot\frac{8}{17} - \frac{9}{41}\cdot\frac{15}{17} = \frac{185}{697}.$$

Method 2. Combining inverse tangents. Suppose we used that $\alpha = 2\tan^{-1}\left(\frac{5}{4}\right)$ and the similar expression for β. This implies that

$$\sin(\alpha+\beta) = \sin\left(2\left[\tan^{-1}\left(\frac{5}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)\right]\right).$$

Let $\theta = \tan^{-1}\left(\frac{5}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$ taking the tangent of both sides and using the tangent sum formula we get $\tan\theta = \frac{\frac{5}{4}+\frac{3}{5}}{1-\frac{5}{4}\cdot\frac{3}{5}} = \frac{37}{5}$ so $\theta = \tan^{-1}\left(\frac{37}{5}\right)$. We can then draw the following triangle



By the double angle identity for sine we get $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{37}{\sqrt{1394}}\cdot\frac{5}{\sqrt{1394}} = \frac{\cancel{2}\cdot 185}{\cancel{2}\cdot 697} = \frac{185}{697}$.

19. **A.** We will do this question by rotating the points back onto the original parabola and then all we have to do is solve a system of equations. There are two ways we can rotate the points back, by use of complex numbers or by use of the rotation matrix. Using matrices we need to rotate all the points (x, y) clockwise $\frac{\pi}{4}$ radians or $\frac{7\pi}{4}$ radians counterclockwise. Either way we have the rotation matrix is $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} = \begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2\\ -\sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}$. To rotate $(8\sqrt{2}, 8\sqrt{2})$ back we compute $\begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2\\ -\sqrt{2}/2 & \sqrt{2}/2\end{bmatrix}\begin{bmatrix}8\sqrt{2}\\ 8\sqrt{2}\end{bmatrix} = \begin{bmatrix}16\\ 0\end{bmatrix}$, continuing like this we find $(\sqrt{2}, -\sqrt{2}) \mapsto (0, -2)$ and $(-\sqrt{2}, \sqrt{2}) \mapsto (0, 2)$. So plugging these into the equation $x = ay^2 + by + c$ we have $c = 16$, $4a - 2b + c = 0$, and $4a + 2b + c = 0$. All that is left to do is solve $4a - 2b = -16$ and $4a + 2b = -16$. Adding these equations we get $a = -4$ and $b = 0$ so our equation is $x = 16 - 4y^2$. From this we have $\sqrt{\left|\frac{c}{a}\right|} = \sqrt{|-4|} = 2$. Alternatively if we wanted to rotate by use of complex numbers we would do $e^{-\pi i/4}(8\sqrt{2} + 8\sqrt{2}i) = e^{-\pi i/4}\cdot 16e^{\pi i/4} = 16e^0 = 16 + 0i \mapsto (16, 0)$. We end up with the same result either way.

20. **C.**

I. True since $\tan^{-1} x$ is always between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ exclusive.

II. True because the period will just be the least common multiple of the individual periods. Note that a constant function is periodic by definition.

III. False because you can rotate a function that intersects its horizontal asymptote to get a slant asymptote. For example if we rotate $f(x) = \frac{x-1}{x^2+1}$ 29° counterclockwise about the origin we will get a slant asymptote that intersects the graph.

IV. True because $\cos^{-1} x$ is between 0 and π so $\sin\left(\cos^{-1} x\right)$ will be positive. Similarly, $\cos\left(\sin^{-1} x\right)$ will also always be positive. From the Pythagorean identity we know $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ with the sign depending on where the angle is. In our case, we know that our sine/cosine will always be positive so we can look at just the positive sign.

V. Not always true, you can pick a point outside of the range of arcsine such as $\frac{2\pi}{3}$ so $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \neq \frac{2\pi}{3}$. You can pick a similar point for cosine.

VI. False because secant and cosecant are complementary yet $\csc^{-1} x + \sec^{-1} x = \frac{\pi}{2}$ if and only if x is not between -1 and 1. The statement is not true for every x.

So the only true statements are I, II, and IV.

21. **C.** From the power reducing identities,

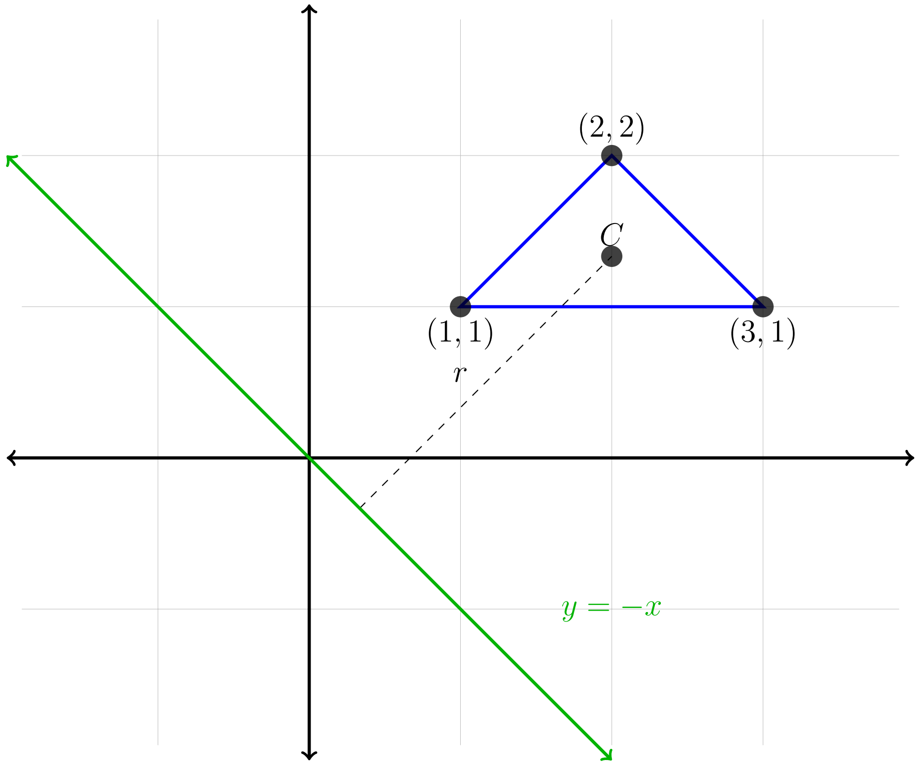
$$\sin^4\left(\frac{\pi}{12}\right) = \left(\frac{1-\cos\left(2\cdot\frac{\pi}{12}\right)}{2}\right)^2.$$

Since $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ we have

$$\left(\frac{1-\frac{\sqrt{3}}{2}}{2}\right)^2 = \left(\frac{2-\sqrt{3}}{4}\right)^2 = \frac{7-4\sqrt{3}}{16}.$$

So $a = \frac{7}{16}, b = -\frac{1}{4}$ so $\frac{a}{b} = -\frac{7}{4}$

22. **B.** We can draw a picture as follows



So first we need the area, this can either be computed by the shoelace theorem or by seeing the height is 1 and the base is 2 so the area is 1. Then we need the centroid which is the average of the x values and the average of the y values. Computing this $\frac{1+2+3}{3} = 2$ and $\frac{1+2+1}{3} = \frac{4}{3}$ so the centroid is labeled above at C which is $\left(2, \frac{4}{3}\right)$. We must now calculate the shortest distance between the centroid and the line. The formula for distance between a point (x_0, y_0) and a line $Ax + By + C = 0$ is

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}.$$

Using this we have our line as $x + y + 0 = 0$ so

$$r = \frac{\left|2 + \frac{4}{3} + 0\right|}{\sqrt{1^2 + 1^2}} = \frac{5\sqrt{2}}{3}.$$

The final answer is $2\pi \cdot \frac{5\sqrt{2}}{3} \cdot 1 = \frac{10\pi\sqrt{2}}{3}$.

23. **C.** If the first factor is 0 then $\sin x = \sqrt{3}\cos x$ which implies $\tan x = \sqrt{3}$. If the second factor is 0 then $\cos x = \frac{1}{2}$. From $\cos x = \frac{1}{2}$ we have $x = \frac{\pi}{6}, \frac{11\pi}{6}$ and from $\tan x = \sqrt{3}$ we have $x = \frac{\pi}{3}, \frac{4\pi}{3}$. So the sum of the solutions is $\frac{17\pi}{3}$ and its coterminal angle is $\frac{5\pi}{3}$.

24. **B.** First we have to ask what the derivative really is. The derivative of a function evaluated at a point x is the slope of the tangent line at the point x. With this in mind, let b be between 0 and 1 exclusive, we have that $f(b) = b - \lfloor b \rfloor = b - 0 = b$. Since this is true for any b between 0 and 1 exclusive we know that $f(x) = x$ if $0 < x < 1$. Now keeping the same b we look at $f(b+k)$ for any positive integer k. Since $0 < b < 1$ we have $k < b+k < 1+k$ so it follows that $\lfloor b+k \rfloor = k$. Thus $f(b+k) = (b+k) - \lfloor b+k \rfloor = b + k - k = b$. So for any non-integer x, $f(x) = x$. Since the problem asks for $f'(a)$ for a non-integer a we are only looking for the slope of the tangent line to $f(x) = x$ which is just 1.

25. **C.** Due to the range of α we know $\cos\alpha < 0$ and similarly $\sin\beta < 0$. Using this we can either draw triangles or recognize that these are Pythagorean triples, either way we obtain $\cos\alpha = -\dfrac{3}{5}$ and $\sin\beta = -\dfrac{12}{13}$. From this we have $\tan\alpha = -\dfrac{4}{3}$ and $\tan\beta = -\dfrac{12}{5}$ and then we can find

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-\frac{4}{3} - \frac{12}{5}}{1 - \frac{4}{3}\cdot\frac{12}{5}} = \frac{-\frac{56}{15}}{-\frac{33}{15}} = \frac{56}{33}.$$

26. **E.** There are 2 main ways to do this question however since this is a precalculus test the way we will do is with trig. Let $x = \cos\theta$ and $y = \sin\theta$, this satisfies the condition $x^2 + y^2 = 1$ so all that is left is that second equation. Plugging this in we have $xy = \cos\theta\sin\theta$ and from the double angle identity this is $\dfrac{1}{2}\sin 2\theta$. This implies

$$\sin 2\theta = \frac{\sqrt{2}}{2} \text{ so } 2\theta = \frac{\pi}{4} + 2\pi k \text{ or } 2\theta = \frac{3\pi}{4} + 2\pi k \text{ for any integer } k.$$

Dividing by 2 we get $\theta = \dfrac{\pi}{8} + \pi k, \dfrac{3\pi}{8} + \pi k$. Going back to the original substitution we get $x = \cos\theta = \cos\left(\dfrac{\pi}{8} + \pi k\right)$, it can be shown that since k is an integer that

$$\cos(\alpha + \pi k) = \cos(\pi k)\cos(\alpha) - \overbrace{\sin(\pi k)\sin(\alpha)}^{0} = \cos(\pi k)\cos(\alpha),$$

plugging in values for k we see that $\cos(\pi k) = (-1)^k$. Keeping this in mind,

$$\cos\left(\frac{\pi}{8} + \pi k\right) = (-1)^k \cos\frac{\pi}{8}$$

and similarly

$$\cos\left(\frac{3\pi}{8} + \pi k\right) = (-1)^k \cos\frac{3\pi}{8}$$

Doing the same thing for $\sin\left(\dfrac{\pi}{8} + \pi k\right)$ we can show that

$$\sin(\alpha+\pi k)=\sin\alpha\cos(\pi k)+\cos\alpha\sin\pi k=(-1)^k\sin\alpha.$$

So $\sin\left(\frac{\pi}{8}+\pi k\right)=(-1)^k\sin\frac{\pi}{8}$ and $\sin\left(\frac{3\pi}{8}+\pi k\right)=(-1)^k\sin\frac{3\pi}{8}$. So if $x=(-1)^k\cos\frac{\pi}{8}$ then $y=(-1)^k\sin\frac{\pi}{8}$, similarly $x=(-1)^k\cos\frac{3\pi}{8}, y=(-1)^k\sin\frac{3\pi}{8}$. Now we have our solution set in terms of k however we want just a general solution. All that the $(-1)^k$ tells us is that the sign of x and y have to be the same so the solution set is

$$\left\{\left(\cos\frac{\pi}{8},\sin\frac{\pi}{8}\right),\left(-\cos\frac{\pi}{8},-\sin\frac{\pi}{8}\right),\left(\cos\frac{3\pi}{8},\sin\frac{3\pi}{8}\right),\left(-\cos\frac{3\pi}{8},-\sin\frac{3\pi}{8}\right)\right\}$$

So from the half angle identities we get $\cos\left(\frac{\pi/4}{2}\right)=\sqrt{\frac{1+\cos\left(\frac{\pi}{4}\right)}{2}}=\frac{\sqrt{2+\sqrt{2}}}{2}$, $\sin\frac{\pi}{8}=\frac{\sqrt{2-\sqrt{2}}}{2}$, $\cos\frac{3\pi}{8}=\frac{\sqrt{2-\sqrt{2}}}{2}$, and $\sin\frac{3\pi}{8}=\frac{\sqrt{2+\sqrt{2}}}{2}$. None of the answers fully covers the solution set so the answer is **E**. Alternatively, you could have solved for y in the second equation to get $y=\frac{\sqrt{2}}{4x}$ and then plug that into the first equation but that doesn't involve any trig and is much less of an interesting way to solve the question.

27. **D.** The factor of $\sqrt{2}$ on the top and the bottom should give you a hint as to what to do with the problem, we can divide both top and bottom inside the square root by $\sqrt{2}$ to get

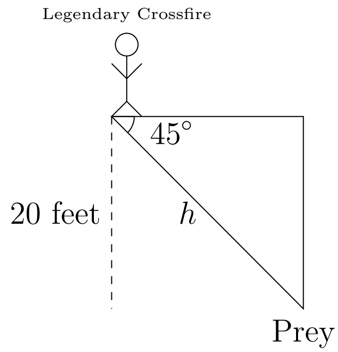
$$\frac{\frac{\sqrt{2}}{2}+\sin\theta}{\frac{\sqrt{2}}{2}+\cos\theta}=\frac{\sin\frac{\pi}{4}+\sin\theta}{\cos\frac{\pi}{4}+\cos\theta}.$$

Now by use of the sum to product identities we get

$$\frac{\cancel{2}\sin\left(\frac{\theta}{2}+\frac{\pi}{8}\right)\cancel{\cos\left(\frac{\theta}{2}-\frac{\pi}{8}\right)}}{\cancel{2}\cos\left(\frac{\theta}{2}+\frac{\pi}{8}\right)\cancel{\cos\left(\frac{\theta}{2}-\frac{\pi}{8}\right)}}=\tan\left(\frac{\theta}{2}+\frac{\pi}{8}\right).$$

Due to the periodicity of tangent we have to take into consideration that $\tan\left(\frac{\theta}{2}+\frac{\pi}{8}\right)=\tan\left(\frac{\theta}{2}+\frac{\pi}{8}+\pi k\right)$ for any integer k and that is why the initial condition is given. Since $f(0)=\frac{\pi}{8}$ the only value for k that works is 0 so $f(\theta)=\frac{\theta}{2}+\frac{\pi}{8}$ and $f\left(-\frac{\pi}{4}\right)=\frac{\pi}{8}$. Alternatively, if you could guess that $f(\theta)$ was a linear function you could guess and check values of θ and arrive at the same answer. Note: You could have chosen any value in the beginning since the solution to $\sin x=\frac{\sqrt{2}}{2}$ has infinitely many solutions, same for $\cos x$, however the problem does not work out unless you pick the same value for x in the top and the bottom. Since there are initial conditions for f it does not matter the angle you pick for example if you picked $x=\frac{9\pi}{4}$ you would still get the same $f(\theta)$ just the value for k would change. Disputes saying that there are infinitely many answers should not be accepted.

28. **C.** We can draw Crossfire's position as follows



The shortest distance will be the length of the hypotenuse (h) of the triangle which can be expressed in terms of $\sin 45°$ as $\dfrac{20}{h}$ so $h = 20 \csc 45° = 20\sqrt{2}$.

29. **A.** The discriminant of B is $1 - 4(1)(-1) > 0$ so it is a rotated ellipse. In C you can solve for y to get $y = \dfrac{x}{x-1}$ which is not two lines. D is just a rotated hyperbola. All that remains is A. To see that this is in fact two lines we can rewrite the equation as $(y-x)^2 = 1$ and we find $y = x \pm 1$.

30. **D.** We have $\ln \dfrac{3}{2} = \ln 3 - \ln 2$ and $\ln 6 = \ln 3 + \ln 2$ so we can rewrite the fraction as $\dfrac{(\ln 3 - \ln 2)(\ln 3 + \ln 2)}{1 - (\ln 3 + \ln 2) + \ln 3 \ln 2} = \dfrac{(a-b)(a+b)}{(1-(a+b)+ab)}$. Looking at the denominator $1 - a - b + ab = (1-a) - b(1-a) = (1-a)(1-b)$ so the final answer is $\dfrac{a^2 - b^2}{(1-a)(1-b)}$