Solutions

Q	A	B	C	D	Answer
1	9	10	2	1	22
2	$\frac{\pi}{2}$	$\frac{1}{3}$	1	1	$\frac{17}{6}$
3	32	89	$\frac{2}{3}$	$\frac{2}{3}$	-57
4	Not	needed	for	compile	$5e^2\ln 2$
5	3	$\frac{7\pi}{6}-\frac{\sqrt{3}}{2}$	$\frac{\pi}{12}$	$\frac{\pi-1}{2}$	17
6	2	-2	-2	-4	6
7	2	$\frac{\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{4}{5}$	$\frac{39}{20}$
8	-35	4	e	e^e	$e-30$
9	2	6	$\ln 2$	$\ln 2$	$8+2\ln 2$
10	$A+$	$B=-1$	$e-1$	0	$e-2$
11	$\frac{25}{28}$	22	$\frac{\pi}{3\sqrt{3}}$	$\ln 23$	$-\frac{\pi\sqrt{3}}{84}$
12	4	$\frac{97}{2}$	250	0	$\frac{605}{2}$
13	276	0	$\ln\left(\frac{4}{3}\right)$	37449	368
14	16	12	14	48	129024

1) **A**: 2^x becomes asymptotically small compared to 3^x as x grows large, so the limit equals $\lim_{x\to\infty}(3^x)^{\frac{2}{x}}=9$.
B: The constants of each function in a radical become asymptotically irrelevant to the value of the limit, so it equals $\lim_{x\to\infty}(|x^2+5|-|x^2-5|)=10$.
C: The limit equals $\lim_{x\to-1}\frac{(x+3)\sin(x^2+4x+3)}{x^2+4x+3}=\lim_{x\to-1}(x+3)=2$.
D: $\ln L=\lim_{x\to0^+}x\ln x=\lim_{x\to0^+}\frac{\ln x}{1/x}=\lim_{x\to0^+}\frac{1/x}{-1/x^2}=\lim_{x\to0^+}-x=0$. Exponentiation gives $L=1$.

$9+10+2+1=\boxed{22}$

2) **A**: $\int_0^\infty\frac{dx}{x^2+1}=\arctan x]_0^\infty=\frac{\pi}{2}$.
B: $\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=\frac{1}{3}$.
C: $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n^2+n}}<\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n^2+k}}<\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n^2}}$. By the Squeeze Theorem, the desired limit is 1.
D: $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2+n}<\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2+k}<\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2}$. By the Squeeze Theorem, the desired limit is 1.

$\frac{1}{2}+\frac{1}{3}+1+1=\boxed{\frac{17}{6}}$

3) **A**: $a(t) = 12t - 4$ so $a(3) = 32$.
B: $x(t) = 2t^3 - 2t^2 - 2t + C$; inputting $(2, 5)$ gives $C = 1$. $x(4) = 89$.
C: $v(t) = 2(3t + 1)(t - 1)$. This has the opposite sign as $a(t)$ when $\frac{1}{3} < t < 1$, an interval with length $\frac{2}{3}$.
D: By Vieta's, the sum of the roots of $6t^2 - 4t - 2 = -2.022$ is $\frac{2}{3}$.

$$32 - 89 + \frac{2}{3} - \frac{2}{3} = \boxed{-57}$$

4) Summing all of the integrals yields the result of the Product Rule applied to $f(x) = e^x(x^2 + 1)\ln x$. $f(2) - f(1) = 5e^2 \ln 2 - 0 = \boxed{5e^2 \ln 2}$.

5) **A**: The area of the lemniscate $r^2 = a^2 \sin(2\theta)$ is a^2, so this area is 3.
B: These curves intersect at $\theta = \pm\frac{\pi}{6}$. The area contained within $r = 2\cos\theta$ but not $r = \sqrt{3}$ is $\displaystyle\int_0^{\frac{\pi}{6}} (4\cos^2\theta - 3)\ d\theta = \int_0^{\frac{\pi}{6}} (2\cos(2\theta) - 1)\ d\theta = \sin(2\theta) - \theta\Big]_0^{\frac{\pi}{6}} = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$. Subtracting this from rest of the area of $r = 2\cos\theta$ gives the desired area of $\frac{7\pi}{6} - \frac{\sqrt{3}}{2}$.
C: $\displaystyle\int_0^{\frac{\pi}{6}} \cos^2(3\theta)\ d\theta = \frac{\theta}{2} + \frac{\sin(6\theta)}{12}\Big]_0^{\frac{\pi}{6}} = \frac{\pi}{12}$.
D: $\displaystyle\frac{1}{2}\int_0^{\ln\pi} e^\theta\ d\theta = \frac{e^\theta}{2}\Big]_0^{\ln\pi} = \frac{\pi - 1}{2}$.

$$3 + \frac{7\pi}{6} - \frac{\sqrt{3}}{2} + \frac{\pi}{12} + \frac{\pi - 1}{2} = \frac{5}{2} - \frac{\sqrt{3}}{2} + \frac{7\pi}{4}.\ 5 - 2 + 14 = \boxed{17}$$

6) $f''(x) = 12x - 4$, $f'(x) = 6x^2 - 4x - 2$, and $f(x) = 2x^3 - 2x^2 - 2x - 4$. $2f(2) - f(1) = -f(1) = \boxed{6}$

7) **A**: The function changes concavity whenever it enters a new quadrant, which happens 2 times.
B: This is a 90° sector cut out of a triangle with radius 1, so its area is $\frac{\pi}{4}$.
C: When $x = -1$ $y = \sqrt{2}$. $|x| = -x$ and $|y| = y$, so $y^2 - x^2 = 1$. $\frac{dy}{dx} = \frac{x}{y}$, which evaluated at the point is $-\frac{1}{\sqrt{2}}$.
D: $\frac{dy}{dx} = -\frac{2x+y}{x+2y}$, which evaluated at the point is $-\frac{4}{5}$.

$$2 + \frac{1}{4} + \frac{1}{2} - \frac{4}{5} = \boxed{\frac{39}{20}}$$

8) **A**: When $y = 1$, $\frac{dy}{dx} = 6$, so $f(2023) = 7$. When $y = 7$, $\frac{dy}{dx} = -42$, so $f(2024) = -35$.
B: This is the logistic differential equation, which will reach stability at $\frac{8}{2} = 4$.
C: $g(1) = e$. Deriving, $g'(x) = \frac{1}{x} - \ln x + g(x)$. Plugging in $x = 1$ gives $g'(1) = 1 + e$.
D: Rearranging, $g' - g = \frac{1}{x} - \ln x$. Multiplying both sides by e^{-x} yields $(ge^{-x})' = (e^{-x}\ln x)'$. Integrating yields $g = e^x + \ln x$ and $g(e) = e^e + 1$.

$$-35 + 4 + 1 + e = \boxed{e - 30}.$$

9) **A**: Computing the derivative of $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ and evaluating at $x = \frac{1}{2}$ yields a sum of 2.
B: Multiplying by x and computing the derivative of this function again, once again evaluating at $x = \frac{1}{2}$ yields a sum of 6.
C: This is the series representation of $\ln(1-x)$ evaluated at $x = -1$, so the sum is $\ln 2$.
D: This is the series representation of e^x evaluated at $x = \ln(\ln 2)$, so the sum is $\ln 2$.

$2 + 6 + \ln 2 + \ln 2 = \boxed{8 + 2\ln 2}$

10) $f'(x) = -\sin x - \int_0^x f(t)\, dt$, and $f''(x) = -\cos x - f(x)$, so $f(x) + f''(x) = -\cos x$. Thus, $\mathbf{A} + \mathbf{B} = -1$.

$g(x) = \lim_{n\to\infty} \int_0^1 \frac{e^x}{1+x^n}\, dx + \lim_{n\to\infty} \int_1^a \frac{e^x}{1+x^n}\, dx = \int_0^1 e^x\, dx = e - 1$. Thus, $g''(x) = 0$.
$-1 + e - 1 + 0 = \boxed{e - 2}$

11) **A**: The integrand is equal to $x^6 - x^3 + 1$, so the integral equals $\frac{1}{7} - \frac{1}{4} + 1 = \frac{25}{28}$.
B: The integral is a well-known demonstration that $\pi < \frac{22}{7}$, and can be shown as such through polynomial long division.
C: $x = u\sqrt{3}$ and $dx = du\sqrt{3}$ give $\frac{1}{\sqrt{3}} \int_0^{\frac{1}{\sqrt{3}}} \frac{du}{u^2+1} = \frac{\pi}{6\sqrt{3}}$.
D: Factoring the integrand gives $\frac{(x+2)(4x+3)}{(x+2)(2x^2+3x+2)}$. Letting $u = 2x^2 + 3x + 2$ gives $\int_2^{46} \frac{du}{u} = \ln 23$.

$\frac{\pi\sqrt{3}}{18}\left(\frac{25}{28} + 22 - 23\right) = \boxed{-\frac{\pi\sqrt{3}}{168}}$

12) **B**: $f(0) + f(1) + f(2) + f(3) = -7 - 4 + 1 + 14 = 4$.
A: $\frac{1}{2}\left(f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2)\right) = \frac{1}{2}\left(\frac{7}{2} + 9 + \frac{51}{2} + 59\right) = \frac{97}{2}$.
S: $\frac{1}{2}(f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)) = \frac{1}{2}(2 + 2 + 56 + 202 + 238) = 250$.
H: Simpson's approximation is exact for cubics, so this is 0.

$0 + \frac{97}{2} + 4 + 250 = \boxed{\frac{605}{2}}$

13) **A**: $f(23, x) = 1 + x + x^2 + \cdots + x^23$, so $f'(23, x)|_{x=1} = 1 + 2 + \cdots + 23 = \frac{23\cdot 24}{2} = 276$.
B: This is the derivative of a constant, which is 0.
C: $\lim_{n\to\infty} f(n, x) = \frac{1}{1-x}$ for $|x| < 1$. Integrating $\int_{\frac{1}{3}}^{\frac{1}{2}} \frac{dx}{1-x} = -\ln|1-x|]_{\frac{1}{3}}^{\frac{1}{2}} = \ln 2 - \ln\frac{3}{2} = \ln\frac{4}{3}$.
D: $f(1, 4) = 4^0 + 4^1 = 5$. $f(2, 3) = 3^0 + 3^1 + 3^2 = 13$. $f(5, 8) = 8^0 + 8^1 + 8^2 + 8^3 + 8^4 + 8^5 =$

$1 + 8 + 64 + 512 + 4096 + 32768 = 37449.$

$276 \cdot \frac{4}{3} - 0 \cdot 37449 = \boxed{368}$

14) **A**: 16
B: 12
C: 14
D: 48

$16 \cdot 12 \cdot 14 \cdot 48 = \boxed{129024}$