
CALCULUS MARCH REGIONAL TEST

2022

FLORIDA ASSOCIATION OF MU ALPHA THETA
FAMAT

Throughout this test, you be asked to find $x \pmod{n}$ where x is some real number and $n \in \mathbb{N}$ (n is a natural number). To be clear, $x \pmod{n}$ means the remainder when x is divided by n . For example,

$$\begin{aligned} 17 \pmod{6} &\equiv 5 & 13 \pmod{6} &\equiv 1 \\ 16 \pmod{6} &\equiv 4 & 12 \pmod{6} &\equiv 0 \\ 15 \pmod{6} &\equiv 3 & 11 \pmod{6} &\equiv 5 \\ 14 \pmod{6} &\equiv 2 & 10 \pmod{6} &\equiv 4 \end{aligned}$$

Let $\lfloor x \rfloor$ = the greatest integer less than or equal to x and $\{x\} = x - \lfloor x \rfloor$.

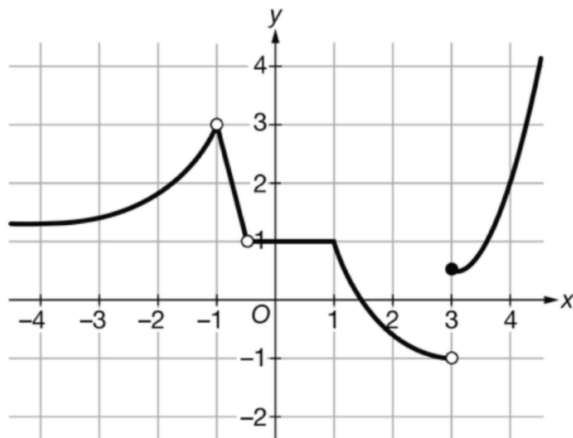
Select answer choice **(E)** if **None Of The** provided **Answers** are correct.

1. Consider the curve defined by $xy^2 - 2x^3 = 1$ given $y \geq 0$ while $D = \frac{d^2y}{dx^2}$. If $(x, y) = (1, 2)$, then $8D \pmod{6} \equiv$ 1. _____
(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

2. Let $\frac{df}{dx} = \sin \pi x + x^2 + 3$, $f(0) = 2$, and $g(x) = \frac{1}{f(x)}$. Find the tangent line to $g(x)$ at $x = 0$. 2. _____
 If the tangent line to $g(x)$ is written in the form $y = mx + b$ then $4(b - m) \pmod{6} \equiv$
(A) 2 (B) 3 (C) 4 (D) 5 (E) [NOTA]

3. If one was completing an $\epsilon - \delta$ proof regarding the $\lim_{x \rightarrow 2} (3x - 1) = 5$, given $\epsilon > 0$, which of the following choices is the largest δ that can be selected? 3. _____
(A) $\delta = \frac{\epsilon}{6}$ (B) $\delta = .25\epsilon$ (C) $\delta = .5\epsilon$ (D) $\delta = 3\epsilon$ (E) $\delta = 5\epsilon$ (F) **NOTA**

4. If $f(1.5) = 0$ and $f(x)$ has the graph provided below, 4. ____



Graph of f

$$\frac{\lim_{x \rightarrow 1.5} (f(2x)f(x))}{\lim_{x \rightarrow 1.5^+} (f(f(2x)))} =$$

- (A) DNE (B) -1 (C) 0 (D) 1 (E) NOTA

5. Let n be the number of points where $f(x)$ is discontinuous if $f(x) = \left\{ \frac{x}{5} \right\} + \left\lfloor \frac{x}{2} \right\rfloor, x \in [0, 100]$. 5. ____
 $n \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

6. If $f'(x) = \sin(x^2)$ and $y = f(x^2 + 1)$, let $\left. \frac{df}{dx} \right|_{x=1} = a \sin b, b \in \left[\pi, \frac{3\pi}{2} \right]$. 6. ____
 $a + b \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

7. If $f(x) = \cos(x^2 - 4[x])$, let $\frac{a}{b} = \frac{1}{\pi} \left(f' \left(\frac{\sqrt{\pi}}{2} \right) \right)^2$ where a and b are coprime positive integers. 7. ____
 $a + b \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

8. If $f(x) = x^3 + 4x^2 + 6x$, let $\left. \frac{df^{-1}(x)}{dx} \right|_{x=-4} = \frac{a}{b}$ where a and b are coprime positive integers. 8. ____
 $b^3 - a \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

9. If $y = \arctan \left[\frac{xa^{\frac{2}{3}} - ax^{\frac{2}{3}}}{(ax)^{\frac{2}{3}} + ax} \right]$ and $\left. \frac{dy}{dx} \right|_{x=27} = \frac{a}{b}$ where $a, b \in \mathbb{N}$ and $\gcd(a, b) = 1$. 9. ____
 $a + \frac{b}{272} \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

10. If $y = \arctan \left(\frac{x}{1 + \sqrt{1 - x^2}} \right)$, $|x| \leq 1$, let $\left. \sqrt{\frac{a}{b}} = \frac{dy}{dx} \right|_{x=\frac{1}{2}}$ where $a, b \in \mathbb{N}$ and $\gcd(a, b) = 1$. 10. ____
 $a + b \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

11. If differentiable $f(x)$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$, $f(5) = -2$, $f'(0) = 3$, then 11. ____
 $\frac{f'(5)}{2} \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

12. $\frac{S}{2022} = \lim_{n \rightarrow \infty} \sum_{k=1}^{\frac{3n}{2}} \frac{\pi}{2n} \sin \left(\frac{k\pi}{n} \right)$ 12. ____

$S \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

13. If $\left(1 + \frac{1}{n} \right)^{n+X_n} = e$ for all positive integers n , let $\lim_{n \rightarrow \infty} X_n = \frac{a}{b}$ where a and b are relatively prime positive integers, then 13. ____
 $a + b \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

14. $136 \cdot \lim_{x \rightarrow 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x} \pmod{6} \equiv$ 14. ____

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

15. If $x = y + \frac{1}{y + \frac{1}{y + \frac{1}{y + \dots}}}$, let $\frac{dy}{dx} = ax^2 + by^2 - cxy$ where $a, b,$ and c are coprime positive integers, 15. ____

$a + 3b + 2c \pmod{6} \equiv$

- 2 (A) 3 (B) 4 (C) 5 (D) NOTA

16. If (x_i, y_i) are the points of intersection of $x^2 = 4y$ and all possible normal lines to the parabola $x^2 = 4y$ going through the point $(1, 2)$ excluding the point $(1, 2)$, then 16. ____

$\sum_i (x_i^2 + y_i^2) \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

17. If $f(x) = Ax^3 + Bx^2 + C$, has a point of extrema at $(2, 11)$ and an inflection point at $(1, 5)$, $|C|^2 (|A + 1|^2 + |B + 1|^2) \pmod{6} \equiv$ 17. ____

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

18. A point moves along the x -axis such that its velocity at time t where $t > 0$, is $v(t) = \frac{\ln t}{t}$. At what value of $\ln t$ is v maximized? 18. ____

- (A) There is no maximum (B) 0 (C) .5 (D) 1 (E) NOTA

19. $\left(\int_{-\infty}^{\infty} \frac{1}{\pi(e^x + e^{-x})} dx \right)^{-1} \pmod{6} \equiv$ 19. ____

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

20. Assuming $31 < \pi^3 < 32$, what is $\left[\left(\int_0^1 x^{-\frac{1}{2}}(1-x)^{-\frac{1}{2}} dx \right)^3 \right] \pmod{6} \equiv$ 20. _____

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

21. If A, B, C, D are coprime integers, $\int_1^\infty \frac{2x^3 - 1}{x^6 + 2x^3 + 9x^2 + 1} dx = \frac{\pi}{6} - \frac{A}{B} \arctan\left(\frac{C}{D}\right)$ 21. _____
 $A + B^2 + C^3 + D^4 \pmod{6} \equiv$

- (A) 2 (B) 2 (C) 4 (D) 5 (E) NOTA

22. Let $I = \int_0^{\frac{\pi}{2}} x \left| (\sin x)^2 - \frac{1}{2} \right| dx$. 22. _____
 $\frac{104I}{\pi} \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

23. $1 + \left(\frac{1}{10\pi^2} \int_0^5 \frac{\arctan(x - \lfloor x \rfloor)}{1 + (x - \lfloor x \rfloor)^2} dx \right)^{-1} \pmod{6} \equiv$ 23. _____

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

24. Let $I_n = \int_0^{2n\pi} \max\{\sin x, \arcsin(\sin x)\} dx$. If $a, b \in \mathbb{Z}^+$, and $\frac{dI_n}{dn} \Big|_{n=2022} = \frac{\pi^2 - a}{b}$, 24. _____
 $1 + b^a \pmod{6} \equiv$

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

25. $\lim_{x \rightarrow 0} \left(\frac{e^{2x} - 1}{\tan x} \right) \pmod{6} \equiv$ 25. _____

- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

26. What does $f'(1)$ equal if $f(x) = (x^2 + 1)^{(2-3x)}$ 26. ____
(A) $-.5 \ln(8e)$ (B) $-.5 \ln(8e)$ (C) $-1.5 \ln 2$ (D) $-.5$ (E) *NOTA*
27. Given $a, b \in \mathbb{Z}^+$, $\gcd(a, b) = 1$, and the area bounded by $f(x) = \frac{x^3}{3} - x^2 + c$, $x = 0$, $x = 2$, and the x -axis is a minimum. If $\frac{a}{b} = c$, $a + b \pmod{6} \equiv$ 27. ____
(A) 2 (B) 3 (C) 4 (D) 5 (E) *NOTA*
28. For what value of k will $x + \frac{k}{x}$ have a relative minimum at $x = -2$? 28. ____
(A) 2 (B) 3 (C) 4 (D) 5 (E) *NOTA*
29. Let $S = \int_{-1}^3 \sqrt{1 + a^2 - 2a} da$ 29. ____
 $S \pmod{6} \equiv$
(A) 2 (B) 3 (C) 4 (D) 5 (E) *NOTA*
30. Let n be the number of distinct letters in American Heritage Palm Beach. 30. ____
 $n \pmod{6} \equiv$
(A) 2 (B) 3 (C) 4 (D) 5 (E) *NOTA*