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Gaither-Leto Invitational
PreCalculus Individual
January 27, 1996

IF NONE OF THE ANSWERS GIVEN IS CORRECT CHOOSE NOTA.

1. Find the radian measure of the smaller angle between the hands of a clock at 2:30.

- a) $\frac{7\pi}{12}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{3}$ d) $\frac{11\pi}{18}$ e) Nota

2. Simplify. $\frac{\frac{2}{x+y} - \frac{1}{x-y}}{\frac{4(x-y)}{x+y} - \frac{x+y}{x-y}}$

- a) $\frac{x+3y}{(3x-y)(x-3y)}$ b) $\frac{3y^2-10xy}{(3x-y)(x-y)}$ c) $\frac{x+3y}{3x-y}$ d) $\frac{1}{3x-y}$ e) Nota

3. Find the intersection of the domain and range over the Reals of $y = 2\sqrt{4x^2-1}$.

- a) $|R| \geq \frac{1}{2}$ b) $R \geq 1$ c) $R \geq \frac{1}{2}$ d) $R \geq 2$ e) Nota

4. For what values of k are $2k$, $5k + 2$, and $20k - 4$ consecutive terms of a geometric sequence?

- a) 2 b) $\frac{2}{5}, 2$ c) $-2, \frac{2}{15}$ d) $\frac{1}{2}, 15$ e) Nota

5. Find the number of possible triangles if in $\triangle ABC$, $a = 176$, $b = 189$, and $m\angle A = 76.4^\circ$.

- a) none b) one c) two d) can't be determined e) Nota

6. Find $a + b + c$ if the parabola determined by $y = ax^2 + bx + c$ passes through the points $(-1, 5)$, $(6, 4)$, and $(2, -4)$.

- a) $\frac{-1}{7}$ b) $\frac{-8}{7}$ c) $\frac{-17}{7}$ d) $\frac{4}{7}$ e) Nota

7. Solve for x. $y = \ln(x^2 - 1)$.

- a) $\sqrt{e^y + 1}$ b) $\pm\sqrt{e^y + 1}$ c) $\sqrt{e^{y+1}}$ d) $\pm\sqrt{e^{y+1}}$ e) Nota

8. Find $\left(\frac{2+3i}{3-i}\right)^2$.

- a) $\frac{-56+33i}{50}$ b) $\frac{3+11i}{10}$ c) $\frac{-56+33i}{5}$ d) $\frac{65+33i}{50}$ e) Nota

9. The amount of coal used by a steamship traveling at a uniform speed varies jointly as the distance traveled and the square of the speed. If a steamship uses 45 tons of coal traveling 80 mi at 15 knots, how many tons will it use if it travels 120 mi at 20 knots?

- a) 60 b) 120 c) 1,200 d) 4,800 e) Nota

10. Find the value of $\csc\phi$ if ϕ is in quadrant IV and the terminal side of ϕ is parallel to the line $12y + 5x = 13$.

- a) $\frac{-5}{12}$ b) $\frac{-12}{5}$ c) $\frac{-13}{12}$ d) $\frac{-13}{5}$ e) Nota

11. Find the equation in polar form of the conic with focus at the pole, directrix perpendicular to the polar axis and 4 units to the left of the pole, and eccentricity of $\frac{1}{4}$.

- a) $r = \frac{1}{4-\sin\theta}$ b) $r = \frac{4}{4+\sin\theta}$ c) $r = \frac{1}{4+\cos\theta}$ d) $r = \frac{4}{4-\cos\theta}$ e) Nota

12. If α and β are angles in quadrant I and $\sin\alpha = r$ and $\cos\beta = s$, find $\sec(\alpha + \beta)$.

- a) $\frac{1}{s\sqrt{1-r^2} + r\sqrt{1-s^2}}$ b) $\frac{\sqrt{1-r^2} + \sqrt{1-s^2}}{s-r}$ c) $\frac{s\sqrt{1-r^2} + r\sqrt{1-s^2}}{s^2-r^2}$ d) $\frac{sr(\sqrt{1-r^2} + \sqrt{1-s^2})}{(s+r)(s-r)}$ e) Nota

13. Find the area of the region determined by $x = 0$, $y = 0$, $y = 3x - 3$ and $y = 0.5x + 2$.

- a) $\frac{5\sqrt{10}}{2}$ b) 3.5 c) 5 d) 10 e) Nota

14. The 7 dwarfs are working in the forest. The probabilities of each one doing the most work are $\frac{1}{3}$ for Doc, $\frac{1}{5}$ for Grumpy, $\frac{1}{6}$ for Happy, $\frac{1}{7}$ for Sneezy, $\frac{1}{10}$ for Dopey, and $\frac{1}{30}$ for Bashful. What is the probability that Sleepy will do the most work?

- a) $\frac{2}{35}$ b) $\frac{2}{7}$ c) $\frac{1}{15}$ d) $\frac{5}{42}$ e) Nota

15. $f(n) = \frac{n^n}{n!}$. $\frac{f(n+1)}{f(n)^2} =$

- a) $\left(\frac{n+1}{n}\right)^{n+1}$ b) $\left(\frac{n}{n+1}\right)^n$ c) $\left(1 + \frac{1}{n}\right)^n$ d) $\left(\frac{n+1}{n}\right)^{n+1}$ e) Nota

16. Find an equation for the locus of points such that the sum of the distances of each point from (2,5) and (2,-3) is 14.

a) $\frac{(x-2)^2}{49} + \frac{(y-1)^2}{33} = 1$ b) $\frac{(x-2)^2}{180} + \frac{9(y-1)^2}{196} = 1$ c) $\frac{(x-2)^2}{180} - \frac{(y-1)^2}{196} = 1$ d) $\frac{(x-2)^2}{33} + \frac{(y-1)^2}{49} = 1$ e) Nota

17. Which of the following statements is false? $a > 0, b > 0, x > 0, b \neq 1$

a) $\log_9 2 + \log_9 3 > \log_9 5$ b) $2 + \log_{16} 16 = 5^{\log_5 3}$ c) $(\log_b a)(\log_a b) = 1$
 d) $x^{\log_2(\log_2 x)} = (\log_2 x)^{\log_2 x}$ e) Nota

18. An isosceles triangle whose equal sides meet at an angle of 50° is inscribed in a circle. The base of the triangle intercepts an arc of 6.6 cm. The radius of the circle is closest to which number below?

a) 3.4 b) 3.8 c) 4.3 d) 4.7 e) 14.4

19. If point A = (-1,2) and point B = (7,8), find the point that is $\frac{3}{4}$ of the way from A to B.

a) (5,6.5) b) (3,5) c) (1,3.5) d) (5,6) e) Nota

20. $C(x) = \frac{5^x + 5^{-x}}{2}$, $F(x) = \frac{5^x - 5^{-x}}{2}$. Find $[C(x)]^2 - [F(x)]^2$.

a) 0 b) $\frac{5^{2x} - 5^{-2x}}{4}$ c) $\frac{5^{2x} + 5^{-2x} - 4}{4}$ d) $\frac{2(5^{2x} + 5^{-2x})}{4}$ e) Nota

21. Which term in the expansion of $(e^x + e^{-x})^{10}$ contains $\frac{1}{e^{2x}}$.

a) third b) fifth c) seventh d) ninth e) Nota

22. An oil tanker and a cruise ship leave port at the same time and travel straight line courses at 10 mph and 25 mph respectively. Two hours later they are 40 mi apart. To the nearest tenth what is the angle between their courses?

a) 40.5° b) 46.7° c) 49.5° d) 130.5° e) Nota

23. $\sin(2\text{Arctan}(\sqrt{x}) + \cos(2\text{Arctan}(\sqrt{x})) =$

- a) $\frac{\sqrt{x+x^2}}{1+x^2}$ b) $\frac{2\sqrt{x}-1}{1+x}$ c) $\frac{x+1}{x^2+1}$ d) $\frac{1+2\sqrt{x}-x}{x+1}$ e) Nota

24. A straight rail line between two cities consists of two segments, one 96 km longer than the other. A passenger train averages 60 km/h over the shorter segment, 120 km/h over the longer segment, and 100 km/h for the entire trip. How far apart are the cities?

- a) 32 b) 128 c) 160 d) 480 e) Nota

25. The polonium isotope has a half life of 140 days. If a sample weighs 20 mg initially, how much, to the nearest tenth, will be left after two weeks?

- a) 17.9 b) 18.7 c) 19.3 d) 19.8 e) Nota

26. Find the area of a triangular patio that has sides of 18 ft, 20 ft, and 22 ft to the nearest square foot.

- a) 160 b) 170 c) 1957 d) 28800 e) Nota

27. A building is 16.3 m from a television tower. From the top of the building, the angle of depression to the base of the tower is 43.5° , and the angle of elevation to the top of the tower is 23.8° . Find the height of the tower to the nearest meter.

- a) 18 b) 22 c) 23 d) 25 e) Nota

28. If $u = r(\cos\theta + i\sin\theta)$ which of the following is false?

- a) $\frac{1}{u} = \frac{1}{r}(\cos(-\theta) + i\sin(-\theta))$ b) $-u = r(\cos(\theta+180^\circ) + i\sin(\theta+180^\circ))$
c) $u^2 = r^2(\cos(2\theta) + i\sin(2\theta))$ d) If $\theta = \frac{\pi}{4}$, then $u^8 = 1$. e) Nota

29. In $\triangle ABC$, $AB = 13$, $AC = 15$, and $BC = 14$. A line perpendicular to \overline{BC} divides the interior of the triangle into two regions of equal area. Find the length of the part of the perpendicular that lies within the triangle.

- a) $4\sqrt{7}$ b) $3\sqrt{5}$ c) $5\sqrt{5}$ d) $4\sqrt{6}$ e) Nota

30. If $x > y$, $x^2 + y^2 = 43$ and $xy = -3$, find $x^6 - y^6$.

- a) $12691\sqrt{37}$ b) 476,560 c) $12880\sqrt{37}$ d) 512,302 e) Nota