Algebra 1 Team Round Answers February 2022 Statewide

Question	A	B	C	D
1	6	-6	3	$2\sqrt{3}$
2	-12	5	278	6
3	12	12	1	-22
4	21	6500000	120	6
5	1	0	-4	10
6	150	7	3	7
7	61	17	1/8	17
8	4	2.65	16	36
9	7	21	-5	-1/2 or equivalent
10	7	38	11	14
11	98	45	0	7
12	$3^8 5$	$2^2 3^2 7$	3^{17}	250
13	$x = 3/2$	16	25	15
14	2	1	5	7

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Question 1

A: **6**

We can factor this expression by difference of squares.

$$(x-3)(x+3)=0$$

$$x=3 \text{ or } -3$$

The positive difference between our two solutions is $3-(-3)=6$.

B: **-6**

$$x^2+12x+36=0$$

$$(x+6)^2=0$$

$$x=-6$$

C: **3**

$$4x^2+8x-5=0$$

$$(2x+5)(2x-1)=0$$

$$x=-\frac{5}{2} \text{ and } \frac{1}{2}$$

The positive difference between our two solutions is $\frac{1}{2}-\left(-\frac{5}{2}\right)=3$.

D: $\mathbf{2\sqrt{3}}$

$$x^2-4x+1=0$$

$$x^2-4x+1+4=4$$

$$(x^2-4x+4)+1=4$$

$$(x-2)^2+1=4$$

$$(x-2)^2=3$$

$$x=2\pm\sqrt{3}$$

The positive difference between our two solutions is $\left(2+\sqrt{3}\right)-\left(2-\sqrt{3}\right)=2\sqrt{3}$.

Question 2

A: **-12**

By first plugging in 3 as x and 3 and y, we get $3*3+3-3*3+2$, which is equal to 5. We then can plug in 2 as x and 5 as y to get $3*2+5-5*5+2$, which is equal to -12.

B: **5**

By plugging in n for x and -1 for y, we get $3*n+(-1)-(-1)^2+2=15$. Simplifying this equation, we have $3n=15$. Dividing both sides of the equation by 3, we have $n=5$.

C: **278**

By first plugging in 4 as a and 5 as b, we get $4 * 4 * 5 - 2 * 5$, which is equal to 70. We then can plug in 70 as a and 1 as b to get $4 * 70 * 1 - 2 * 1$, which is equal to 278.

D: **6**

By plugging in 5 as a and m as b, we get $4 * 5 * m - 2 * m = 108$. Simplifying, we get $20m - 2m = 18m = 108$. Dividing both sides by 18, we have $m = 6$.

Question 3

A: **12**

The x^2y term is $\binom{3}{1}(2x)^2(y)^1$. The coefficient is 3*4, or 12.

B: **12**

For the polynomial $ax^2 + bx + c$, the discriminant is $b^2 - 4ac$. Since a = 2, b = 10, and c = 11, we get that the discriminant is $100 - 88 = 12$.

C: **1**

Adding 25 to both sides, we get $(z + 4)^2 = 25$. Taking the square root, we get that $z + 4 = \pm 5$, where $z = 1, -9$. We want the positive z, so the answer is 1.

D: **-22**

First, to expand K, we can factor out $(9x + y)$ to get $K = (9x + y)(2x + y + x - 3y) = (9x + y)(3x - 2y)$. Expanding this, we get $K = 27x^2 - 15xy - 2y^2$. Next, to expand H, we get $H = 4(x^2 - 6xy + 9y^2) + 18xy - 2y^2 = 4x^2 - 24xy + 36y^2 + 18xy - 2y^2 = 4x^2 - 6xy + 34y^2$. We then subtract $K - H$, to get $27x^2 - 15xy - 2y^2 - (4x^2 - 6xy + 34y^2) = 27x^2 - 15xy - 2y^2 - 4x^2 + 6xy - 34y^2 = 23x^2 - 9xy - 36y^2$. The sum of these coefficients is $23 + (-9) + (-36)$, which is equal to -22.

Question 4

A: **21**

The prime numbers less than 75 include 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, and 73, for a total of 21 primes.

B: **6500000**

Since the first two characters must be distinct letters, and there are 26 total letters, there are 26 options for the first character and 25 options for the second character. The last four characters can be any digit, so there are 10 options for the third character, 10 options for the fourth, 10 options for the fifth, and 10 options for the sixth character. This allows for 26*25*10*10*10*10 = 6500000 total options.

C: **120**

Helena has 5 options to choose for her biology book, 3 options for her chemistry book, and 8 options for her physics book, which means that she has 5*3*8 = 120 options.

D: **6**

Since Kira's four-digit number has to be divisible by 2, meaning even, the last digit cannot be 1, 7, or 9, meaning it has to be 2. Since she can only use each digit once, she has 3 options for her first digit (1, 7, or 9), 2 options for her second digit, and 1 option for her third digit. This allows for 3*2*1*1 = 6 four-digit numbers.

Question 5

A: **1**

$$x^2 - 6x + 5 = 0$$
$$(x-1)(x-5) = 0$$
$$x = 1, 5$$

The smaller solution is 1.

B: **0**

$$x^2 + 4x + 2 = 0$$
$$x^2 + 4x + 2 + 4 = 4$$
$$(x^2 + 4x + 4) + 2 = 4$$
$$(x+2)^2 - 2 = 0$$

The value of $a + b$ is thus $2 + -2 = 0$.

C: **-4**

$$x^2 + 8x + 8 = 0$$
$$x^2 + 8x + 8 + 16 = 16$$
$$(x^2 + 8x + 16) + 8 = 16$$
$$(x+4)^2 - 8 = 0$$

The value of $a + b$ is thus $4 + -8 = -4$.

D: **10**

$$2x^2 + 12x + 23 = 0$$
$$2(x^2 + 6x) + 23 = 0$$
$$2(x^2 + 6x + 9) + 23 - 2(9) = 0$$
$$2(x+3)^2 + 5 = 0$$

The value of $a + b + c$ is thus $2 + 3 + 5 = 10$.

Question 6

A: **150**

If x is the cost of one apple and y is the cost of one orange, we can set up the following system of equations:

$$\begin{cases} x + y = 200 \\ 2x + 4y = 500 \end{cases}$$

If we divide $2x + 4y = 500$ by 2 on both sides, we get that $x + 2y = 250$. Subtracting $x + y = 200$, we see that $y = 50$.

Substituting back into the first equation, we can see that $x + 50 = 200$, so $x = 150$.

B: **7**

We can multiply the top equation by 4 and the bottom equation by 5 so that the coefficient of n in both equations is 20.

$$4(4m + 5n) = 4(88)$$

$$5(4n - 3m) = 5(27)$$

We can then subtract the two equations.

$$16m + 20n - 20n + 15m = 4(88) - 5(27)$$

$$31m = 217$$

$$m = 7$$

C: **3**

We can multiply the top equation by 5 and the bottom equation by 2 so that the coefficient of y in both equations is -10.

$$5(5x - 2y) = 5(1)$$

$$2(6x - 5y) = 2(9)$$

We can then subtract the two equations.

$$25x - 10y - 12x + 10y = 5(1) - 2(9)$$

$$13x = -13$$

$$x = -1$$

We can then substitute back in to see that

$$5(-1) - 2y = 1$$

$$-2y = 6$$

$$y = -3$$

Therefore, $xy = (-1)(-3) = 3$.

D: **7**

Call the number of nickels n. If Alex has 12 total coins, the number of dimes is $12 - n$. We know one dime is worth $\$0.10$ and one nickel is worth $\$0.05$. We can then set up an equation and solve for n:

$$0.10(12 - n) + 0.05(n) = 0.85$$

$$1.2 - 0.10n + 0.05n = 0.85$$

$$-0.05n = -.35$$

$$n = 7 \text{ nickels}$$

Question 7

A: **61**

The nth term of an arithmetic sequence that has first term a and common difference d is $a + (n - 1)d$. We know that the first term is 1 and the common difference is 4, and we want the 16th term, or when n = 16. Plugging these numbers into the equation, we have $1 + (16 - 1)4 = 61$.

B: **17**

The nth term of an arithmetic sequence that has first term a and common difference d is $a + (n - 1)d$. We know that the first term is 3 and the common difference is 2. Setting this equal to 35, we have $35 = 3 + (n - 1)2$. Solving this one-variable equation, we get n = 17.

C: $\mathbf{\frac{1}{8}}$

We see that the pattern is dividing each previous term by 2. Dividing $\frac{1}{4}$ by 2, we get $\frac{1}{8}$.

D: **17**

We see that all the integers from -15 to 15 cancel out to become 0, which means that the sum at n=16 is 16, and the sum at n=17 is 33, which is the first term in which the sum is at least 30.

Question 8

A: **4**

If we call the original price x, a 20% discount means that you must pay 80% of the cost, so the sale price is $0.8x$.

$$0.8x = \$3.20$$

$$x = \$4.00$$

B: **2.65**

The sale price of the pencil that is discounted by 50% is $0.5(\$5.00) = \$2.50$. The amount of tax you must pay in addition is 6% of that price, or $0.06(\$2.50) = \$0.15$.

The total amount to pay is $\$2.50 + \$0.15 = \$2.65$.

C: **16**

Knowing that strawberry concentration $\times$ amount of mixture $=$ amount of strawberry in mixture, we can make a table to keep track of our calculations!

Concentration	Amount	Total
0.20	80	16
0.80	x	$0.80x$
0.30	$x + 80$	$0.30(x + 80)$

The total amount of strawberry in the first and second rows must add to give the third row, so

$$16 + 0.80x = 0.30(x + 80)$$

$$16 + 0.80x = 0.30x + 24$$

$$0.50x = 8$$

$$x = 16$$

D: **36**

$0.45(80) = \frac{9}{20}(80) = 9(4) = 36.$

Question 9

A: **7**

The slope of the line is $\frac{19-(-5)}{4-(-2)} = 4$. We know the equation of the line can thus be written in the form $y = 4x + b$ such that b is the $y-$ intercept. Since $(-2, -5)$ lies on the line, we know that $-5 = 4(-2) + b$, so $b = 3$. The sum of the slope and y-intercept is $4 + 3 = 7$.

B: **21**

The slope of the line is $\frac{5-2}{-4-1} = -\frac{3}{5}$. Using point-slope form, we can write the equation of the line as $y - 2 = -\frac{3}{5}(x - 1)$.

$$y - 2 = -\frac{3}{5}(x - 1)$$
$$-5(y - 2) = 3(x - 1)$$
$$-5y + 10 = 3x - 3$$
$$3x + 5y = 13$$

C: **-5**

We can do a substitution for $y = -2x - \frac{7}{2}$ into the equation $3x - 2y = -28$.

$$3x - 2\left(-2x - \frac{7}{2}\right) = -28$$
$$3x + 4x + 7 = -28$$
$$7x = -35$$
$$x = -5$$

D: $-\frac{1}{2}$

If the two lines are perpendicular, then the line $y = 6kx + 3$ has slope -3, which is the negative reciprocal of $\frac{1}{3}$. The line is in slope-intercept form, so $6k = -3$. Therefore, $k = -\frac{1}{2}$.

Question 10

A: **7**

$$3(x + 7) = 42$$
$$x + 7 = 14$$
$$x = 7$$

B: **38**

The amount of tests that Andrew can write in 10 hours is $30 \times 10 = 300$ tests. For them to finish, Zach must cover the remaining $680 - 300 = 380$ tests in 10 hours. Therefore, he must write at a speed of $\frac{380}{10} = 38$ tests per hour.

C: **11**

$440 \text{ km} \times \frac{1 \text{ hour}}{40 \text{ km}} = 11$ hours.

D: **14**

The three consecutive even integers can be represented as $n-2, n,$ and $n+2$ for some even integer n.

$$(n-2)+(n)+(n+2)=36$$

$$3n=36$$

$$n=12$$

Therefore, the three integers are 10, 12, and 14, the largest of which is 14.

Question 11

A: **98**

Although this may seem difficult given that we only have 1 equation with two variables, we can multiply both sides of the equation by the LCM of the denominators to get rid of the fractions.

$$\frac{4}{7}x+\frac{3}{2}y=7$$

$$14\left(\frac{4}{7}x+\frac{3}{2}y\right)=14(7)$$

$$4(2)x+3(7)y=98$$

$$8x+21y=98$$

Our answer is thus 98.

B: **45**

$$15\left(\frac{x}{5}+\frac{x}{3}\right)=15(24)$$

$$3x+5x=15(24)$$

$$8x=15(24)$$

$$x=\frac{15(24)}{8}=15(3)=45$$

C: **0**

$$\frac{x^2+6x+8}{x+4}=2$$

$$\frac{(x+4)(x+2)}{x+4}=2$$

$$x+2=2$$

$$x=0$$

Note that although evaluating the expression without cancelling out the $x+4$ term might give you an additional solution of $x=-4$, this is extraneous because the denominator of the initial expression cannot be 0.

D: **7**

After using the distributive property, the left side evaluates to $42+6ck$. In order for all values of k to satisfy the equality, the coefficient of k must be the same on both sides.

$$6c=42$$

$$c=7$$

Question 12

A: $\mathbf{3^8 5}$

There are 5 3^8's, therefore, we can factor it out to get $3^8(1+1+1+1+1) = 3^8 * 5$

B: $\mathbf{2^2 3^2 7}$

Since $36 = 6^2$ *and* $216 = 6^3$, we can write $36^{\frac{3}{2}} + 216^{\frac{2}{3}}$ as $(6)^{2*\frac{3}{2}} + (6)^{3*\frac{2}{3}} = 6^3 + 6^2 = 6^2(6+1) = 2^2 3^2 7$.

C: $\mathbf{3^{17}}$

Since $9 = 3^2$, we can write $(9^2)^3 * 3^5$ as $(3^{2^2})^3 * 3^5 = 3^{4*3}3^5 = 3^{12}3^5 = 3^{12+5} = 3^{17}$.

D: **250**

Since $25 = 5^2$, we can write $25^{(x-1)} = 2$ as $5^{2(x-1)} = 5^{2x-2} = 2$. Since we are looking for 5^{2x+1}, we can see that we can obtain that by simply multiplying 5^{2x-2} by 5^3, or $2 * 5^3 = 2 * 125 = 250$.

Question 13

A: $\boldsymbol{x = \frac{3}{2}}$

The axis of symmetry of f(x) is x = x-coordinate of the vertex. The x-coordinate of the vertex is $-\frac{b}{2a}$, or $\frac{12}{8} = \frac{3}{2}$ for our parabola.

B: **16**

The minimum point of f(x) is the vertex, so we can simply find the y-coordinate of the vertex, which is plugging in the x-coordinate, or $-\frac{b}{2a}$, or $\frac{12}{8} = \frac{3}{2}$, into f(x).

$$f(\tfrac{3}{2}) = 4 * \frac{3}{2} * \frac{3}{2} - 12 * \frac{3}{2} + 25 = 9 - 18 + 25 = 16$$

C: **25**

The y-intercept of f(x) is $f(0) = 4(0)^2 - 12(0) + 25 = 25$.

D: **15**

The vertical shift from f(x) to g(x) is the difference in their x-intercepts. Expanding g(x), we get $4x^2 - 12x + 9 + 31 = 4x^2 - 12x + 40$. The absolute value of the difference in the x-intercepts of f(x) and g(x) is $|40 - 25| = 15$.

Question 14

A: **2**

With the absolute value, we know that $2x + 1$ must either equal 7 or -7. We can first set it equal to 7 and see if we get an integral solution.

$$2x + 1 = 7$$

$$2x = 6$$

$$x = 3$$

We then set it equal to -7 and similarly see if we get an integral solution.

$$2x + 1 = -7$$

$$2x = -8$$

$$x = -4$$

Since 3 and -4 are both integral solutions, our answer is 2.

B: **1**

With the absolute value, we know that $5x + 1$ must either equal 9 or -9. We can first set it equal to 9 and see if we get an integral solution.

$$5x + 1 = 9$$

$$5x = 8$$

$$x = \frac{8}{5}$$

Unfortunately, $\frac{8}{5}$ is not an integral solution. We then set it equal to -9 and similarly see if we get an integral solution.

$$5x + 1 = -9$$

$$5x = -10$$

$$x = -2$$

Since -2 is our only integral solution, our answer is 1.

C: **5**

With the absolute value, we know that $3x + 3$ must either be less than or equal to 8 or greater than or equal to -8.

$$3x + 3 \leq 8$$

$$3x \leq 5$$

$$x \leq \frac{5}{3}$$

Unfortunately, $\frac{8}{5}$ is not an integral solution. We then set it equal to -9 and similarly see if we get an integral solution.

$$3x + 3 \geq -8$$

$$3x \geq -11$$

$$x \geq -\frac{11}{3}$$

The biggest integral solution of x is the biggest integer less than $\frac{5}{3}$, or 1. The smallest integral solution of x is the least integer greater than $-\frac{11}{3}$, or -3. Our integral solutions are therefore -3, -2, -1, 0, and 1, for a total of 5 integral solutions.

D: **7**

Expanding the equation, we have $x^2 - 3x - 10 - x^2 < 0$, or $-3x - 10 < 0$. This means $-3x < 10$, or $x > -\frac{10}{3}$. We know that x has to be less than 4, and the smallest value of x is -3. This means that x is -3, -2, -1, 0, 1, 2, and 3, for a total of 7 solutions of x.