ANSWER KEY:

1. D

2. C

3. D

4. A

5. C

6. B

7. A

8. E

9. B

10. D

11. B

12. C

13. A

14. B

15. C

16. B

17. E

18. C

19. D

20. B

21. A

22. C

23. B

24. A

25. B

~~26. D~~ – THROWN OUT!

27. A

28. B

29. C

30. D

SOLUTIONS:

1. $\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{4-x^2}-2}\right)=\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{4-x^2}-2}\right)\left(\frac{\sqrt{4-x^2}+2}{\sqrt{4-x^2}+2}\right)$

$\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{4-x^2}-2}\right)=\lim\limits_{x\to 0^+}\left(\frac{\sqrt{4-x^2}+2}{4-x^2-4}\right)$

$\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{4-x^2}-2}\right)=\lim\limits_{x\to 0^+}\left(\frac{\sqrt{4-x^2}+2}{-x^2}\right)$

$\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{4-x^2}-2}\right)=\lim\limits_{x\to 0^+}\left(\frac{\sqrt{4-x^2}}{-x^2}\right)+\lim\limits_{x\to 0^+}\left(-\frac{2}{x^2}\right)$

$\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{4-x^2}-2}\right)=\lim\limits_{x\to 0^+}\left(\frac{\sqrt{4-x^2}}{-x^2}\right)-\lim\limits_{x\to 0^+}\left(\frac{2}{x^2}\right)$

$\lim\limits_{x\to 0^+}\left(\frac{\sqrt{4-x^2}}{-x^2}\right)=-\left[\lim\limits_{x\to 0^+}\left(\sqrt{4-x^2}\right)\right]\left[\lim\limits_{x\to 0^+}\left(\frac{1}{x^2}\right)\right]$

$\lim\limits_{x\to 0^+}\left(\frac{\sqrt{4-x^2}}{-x^2}\right)=-[2][\infty]=-\infty$

$\lim\limits_{x\to 0^+}\left(\frac{2}{x^2}\right)=\infty$

$\lim\limits_{x\to 0^+}\left(\frac{1}{\sqrt{4-x^2}-2}\right)=-\infty-\infty=-\infty$

Hence the answer is D

2. $x=h\big(g(x)\big)$

$1=h'\big(g(x)\big)g'(x)$

$h'\big(g(x)\big)=\frac{1}{g'(x)}$

Find the value of x, where $g(x)=1$

$g(x)=1=2x^3+4x^2+2x+5$

$0=2x^3+4x^2+2x+4$

$0=x^2(2x+4)+(2x+4)$

$0=(x^2+1)(2x+4)$

Only real solution is $x=-2$ → $g(-2)=1$

$h'\big(g(-2)\big)=h'(1)=\frac{1}{g'(-2)}$

$g'(x)=6x^2+8x+2$

$g'(-2)=6(-2)^2+8(-2)+2=10$

$h'(1)=\frac{1}{g'(-2)}=\frac{1}{10}$

Hence the answer is C

3. $f(x) = \frac{1}{x(x-1)}$ is not continuous at $x = 0$ and not differentiable at $x = 0$

$f(x) = \ln(x^2)$ is not continuous at $x = 0$ and not differentiable at $x = 0$

$f(x) = \sqrt{x}$ is continuous at $x = 0$ and not differentiable at $x = 0$

$f(x) = \tan(x)$ is continuous at $x = 0$ and is differentiable at $x = 0$

Hence the answer is D

4. $y^2x^4 - x^2y = 4y + 24$

$y^2x^4 = y(4 + x^2) + 24$

$\frac{dy}{dx}2yx^4 + 4y^2x^3 = \frac{dy}{dx}(4 + x^2) + 2xy$

$\frac{dy}{dx} = \frac{2xy - 4y^2x^3}{2yx^4 - (4 + x^2)}$

$\frac{dy}{dx} = \frac{[2(2)(-1)] - [4(-1)^2(2)^3]}{[2(-1)(2)^4] - [4 + (2)^2]}$

$\frac{dy}{dx} = \frac{(-4) - (32)}{(-32) - (8)} = \frac{9}{10}$

Hence the answer is A

5. $\vec{v}(t) = \langle 4t - 5, 6t^2 - 14t \rangle$

$\vec{s}(t) = \langle 2t^2 - 5t + C_1, 2t^3 - 7t^2 + C_2 \rangle$

From the given information: $C_1 = C_2 = 0$

$\vec{s}(t) = \langle 2t^2 - 5t, 2t^3 - 7t^2 \rangle$

At $t = 3$ → $\vec{s}(3) = \langle 2(3)^2 - 5(3), 2(3)^3 - 7(3)^2 \rangle$

$\vec{s}(3) = \langle 3, -9 \rangle$ and $\vec{s}(0) = \langle 0, 0 \rangle$

$d = \sqrt{(3 - 0)^2 + (-9 - 0)^2} = \sqrt{90} = 3\sqrt{10}$

Hence the answer is C

6. $\int_a^b f(x)dx = \left(\frac{b-a}{n}\right)\left(\frac{1}{2}\right)\left(f(x_0) + f(x_1)\right) + \left(\frac{b-a}{n}\right)\left(\frac{1}{2}\right)\left(f(x_1) + f(x_2)\right) + \left(\frac{b-a}{n}\right)\left(\frac{1}{2}\right)\left(f(x_2) + f(x_3)\right)$ for $n = 3$

$\int_0^6 (x^2 - 3x + 3)dx = \left(\frac{6-0}{6}\right)\left(f(0) + f(2)\right) + \left(\frac{6-0}{6}\right)\left(f(2) + f(4)\right) + \left(\frac{6-0}{6}\right)\left(f(4) + f(6)\right)$

$\int_0^6 (x^2 - 3x + 3)dx = \left(\frac{6-0}{6}\right)(3 + 1) + \left(\frac{6-0}{6}\right)(1 + 7) + \left(\frac{6-0}{6}\right)(7 + 21)$

$\int_0^6 (x^2 - 3x + 3)dx = (4) + (8) + (28) = 40$

Hence the answer is B

7. Observe the highest-ordered terms

$$\lim_{x\to\infty}\left(\frac{\sqrt{18x^8}}{3x^4}\right) = \lim_{x\to\infty}\left(\frac{3\sqrt{2}x^4}{3x^4}\right)$$

$$\lim_{x\to\infty}\left(\frac{\sqrt{18x^8}}{3x^4}\right) = \sqrt{2}$$

Hence the answer is A

8. Find the points on intersection $4x + 2y = 12$ and $x^2 = y + 9$

$$x^2 = (6 - 2x) + 9$$

$$x^2 + 2x - 15 = 0$$

$$(x + 5)(x - 3) = 0$$

$$x = \{-5,3\}$$

$$A_{equilaeral\ triangle} = \frac{\sqrt{3}}{4}s^2$$

$$\int_{-5}^{3}\left[\frac{\sqrt{3}}{4}\left(6 - 2x - (x^2 - 9)\right)\right]dx = \frac{\sqrt{3}}{4}\left[\left(15x - x^2 - \frac{1}{3}x^3\right)\Big|_{-5}^{3}\right]$$

$$\int_{-5}^{3}\left[\frac{\sqrt{3}}{4}\left(6 - 2x - (x^2 - 9)\right)\right]dx = \frac{\sqrt{3}}{4}\left[\left(15(3) - (3)^2 - \frac{1}{3}(3)^3\right) - \left(15(-5) - (-5)^2 - \frac{1}{3}(-5)^3\right)\right]$$

$$\int_{-5}^{3}\left[\frac{\sqrt{3}}{4}\left(6 - 2x - (x^2 - 9)\right)\right]dx = \frac{\sqrt{3}}{4}\left[(45 - 9 - 9) - \left(-75 - 25 + \frac{125}{3}\right)\right]$$

$$\int_{-5}^{3}\left[\frac{\sqrt{3}}{4}\left(6 - 2x - (x^2 - 9)\right)\right]dx = \frac{\sqrt{3}}{4}\left[(27) - \left(-100 + \frac{125}{3}\right)\right]$$

$$\int_{-5}^{3}\left[\frac{\sqrt{3}}{4}\left(6 - 2x - (x^2 - 9)\right)\right]dx = \frac{\sqrt{3}}{4}\left[(27) - \left(-\frac{175}{3}\right)\right]$$

$$\int_{-5}^{3}\left[\frac{\sqrt{3}}{4}\left(6 - 2x - (x^2 - 9)\right)\right]dx = \frac{\sqrt{3}}{4}\left[(27) - \left(-\frac{175}{3}\right)\right]$$

$$\int_{-5}^{3}\left[\frac{\sqrt{3}}{4}\left(6 - 2x - (x^2 - 9)\right)\right]dx = \frac{\sqrt{3}}{4}\left[\frac{256}{3}\right] = \frac{64\sqrt{3}}{3}$$

Hence the answer is C

9. $d = 18m, H = 24m$

From the given, $\frac{dV}{dt} = 14\frac{m^3}{hr} - 5\frac{m^3}{hr} = 9\frac{m^3}{hr}$

$V = BH = \left(\frac{1}{2}d\right)^2 \pi H$

The diameter of the tank never changes and is not time dependent.

$V = \left(\frac{1}{2}(18m)\right)^2 \pi H$

$V = 81\pi H\ m^2$

$\frac{dV}{dt} = (81\pi\ m^2)\frac{dH}{dt}$

$\frac{dH}{dt} = \frac{1}{81\pi\ m^2}\frac{dV}{dt}$

$\frac{dH}{dt} = \frac{1}{81\pi m^2}\left(9\frac{m^3}{hr}\right) = \frac{1}{9\pi}\frac{m}{hr}$

Hence the answer is B

10. $\frac{dy}{dx} = \frac{x}{y} \rightarrow ydy = xdx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$\frac{1}{2}y^2 - \frac{1}{2}x^2 = C$

If $C = 0$, the solution is a degenerate conic or a pair of intersecting lines

Hence the answer is D

11. $([x])^2 = 4$ on the interval $1.5 \le x < 2$

$([x])^2 = 9$ on the interval $2 \le x < 3$

….

$([x])^2 = 225$ on the interval $14 \le x < 15$

$([x])^2 = 256$ on the interval $15 \le x < 15.5$

$\int_{1.5}^{15.5}([x])^2\,dx = \frac{1}{2}(4) + \sum_{n=3}^{15}(n^2) + \frac{1}{2}(256)$

$\int_{1.5}^{15.5}([x])^2\,dx = \frac{1}{2}(4) + \left[\sum_{n=1}^{15}(n^2) - 1^2 - 2^2\right] + \frac{1}{2}(256)$

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \rightarrow \int_{1.5}^{15.5}([x])^2\,dx = 2 + \left[\frac{15(16)(31)}{6} - 5\right] + 128$

$\int_{1.5}^{15.5}([x])^2\,dx = 2 + [5(8)(31) - 5] + 128 = 1365$

Hence the answer is B

12. $\lim_{x\to 0^+}[x] = 0$

$\lim_{x\to 0^+}\left[x\cos\left(\frac{2}{\sqrt{x}}\right)\right] = \left(\lim_{x\to 0^+}[x]\right)\left(\lim_{x\to 0^+}\left[\cos\left(\frac{2}{\sqrt{x}}\right)\right]\right)$

$\lim_{x\to 0^+}\left[x\cos\left(\frac{2}{\sqrt{x}}\right)\right] = (0)\left(\lim_{x\to 0^+}\left[\cos\left(\frac{2}{\sqrt{x}}\right)\right]\right) = 0$

Hence the answer is C

13. $\int_1^{27}\frac{dx}{1+\sqrt[3]{x}}$

Use the u-substitution $u = 1+\sqrt[3]{x}$ and $3(u-1)^2 du = dx$

$\int\frac{dx}{1+\sqrt[3]{x}} = \int\frac{3(u-1)^2}{u}du$

$\int\frac{dx}{1+\sqrt[3]{x}} = \int\left(3u-6+\frac{3}{u}\right)du = \frac{3}{2}u^2-6u+3\ln|u|+C$

$\int_1^{27}\frac{dx}{1+\sqrt[3]{x}} = \frac{3}{2}\left(1+\sqrt[3]{x}\right)^2-6\left(1+\sqrt[3]{x}\right)+3\ln\left|1+\sqrt[3]{x}\right|\Big|_1^{27}$

$\int_1^{27}\frac{dx}{1+\sqrt[3]{x}} = \left[\frac{3}{2}(4)^2-6(4)+3\ln|4|\right]-\left[\frac{3}{2}(2)^2-6(2)+3\ln|2|\right]$

$\int_1^{27}\frac{dx}{1+\sqrt[3]{x}} = [24-24+3\ln|4|]-[6-12+3\ln|2|]$

$\int_1^{27}\frac{dx}{1+\sqrt[3]{x}} = 6+6\ln|2|-3\ln|2|$

$\int_1^{27}\frac{dx}{1+\sqrt[3]{x}} = 6+3\ln|2| = 6+\ln|8|$

Hence the answer is A

14. $\int_9^{21}\left(\frac{8}{x^2-2-15}\right)dx = \int_9^{21}\left(\frac{1}{x-5}-\frac{1}{x+3}\right)dx$

$\int_9^{21}\left(\frac{8}{x^2-2-15}\right)dx = (\ln|x-5|-\ln|x+3|)|_9^{21}$

$\int_9^{21}\left(\frac{8}{x^2-2-15}\right)dx = (\ln|16|-\ln|24|)-(\ln|4|-\ln|12|)$

$\int_9^{21}\left(\frac{8}{x^2-2-15}\right)dx = \left(\ln\left|\frac{2}{3}\right|\right)-\left(\ln\left|\frac{1}{3}\right|\right)$

$\int_9^{21}\left(\frac{8}{x^2-2-15}\right)dx = \ln(2)$

Hence the answer is B

15. $f_{avg}=\frac{1}{6-2}\int_2^6(-2x^2-8)dx \rightarrow f_{avg}=\frac{1}{6-2}\left[-\frac{2}{3}x^3-8x\Big|\begin{matrix}6\\2\end{matrix}\right]$

$f_{avg}=\frac{1}{6-2}\left[\left(-\frac{2}{3}(216)-48\right)-\left(-\frac{16}{3}-16\right)\right]$

$f_{avg}=\frac{1}{6-2}\left[(-144-48)-\left(-\frac{16}{3}-16\right)\right]$

$f_{avg}=\left[(-36-12)-\left(-\frac{4}{3}-4\right)\right]=-\frac{128}{3}$

Hence the answer is C

16. $f(x)=x^4-2x^3-12x^2+8x-6$

$f'(x)=4x^3-6x^2-24x+8$

$f''(x)=12x^2-12x-24$

$f''(x)=12(x^2-x-2)=12(x-2)(x+1)$

The second derivative is negative on the interval $(-1,2)$

Hence the answer is B

17. $\pi\int_0^a(f(x)-1)^2dx=64a\pi$

$\int_0^a(f(x)-1)^2dx=64a$

Recall the Second Fundamental Theorem of Calculus: If $f(x)$ is a continuous function on a closed interval I and c is a constant in the interval I, then $\frac{d}{dx}\left(\int_c^x g(t)dt\right)=f(x)dx$

$\frac{d}{da}\int_0^a\left[\left(f(x)\right)^2-2f(x)+1\right]dx=2f(a)f'(a)-2f'(a)+\left[\frac{d}{da}(1)\right]\times f'(a)$

$\frac{d}{da}\int_0^a\left[\left(f(x)\right)^2-2f(x)+1\right]dx=2f(a)f'(a)-2f'(a)$

$2f(a)f'(a)-2f'(a)=\frac{d}{da}(64a)$

$f(a)f'(a)-f'(a)=32$

$f'(a)[f(a)-1]=32$

Integrate both sides with respect to a: $\frac{1}{2}(f(a)-1)^2=32a+c_1$

$f(a)=1\pm\sqrt{64a+c_2}$ → Since $f(x)>1$ for all x: $f(a)=1+\sqrt{64a+c_2}$

$f(0)=5$: $f(0)=5=1+\sqrt{c_2}$

$c_2=16$

For $a=2$: $f(2)=1+\sqrt{64(2)+16}=13$

Hence the answer is E

18. $\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = \int_{\pi/4}^{2\pi/3}\left(\sqrt{1+\cos(x)}\right)\left(\frac{\sqrt{1-\cos(x)}}{\sqrt{1-\cos(x)}}\right)dx$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = \int_{\pi/4}^{2\pi/3}\left(\frac{\sqrt{\sin^2(x)}}{\sqrt{1-\cos(x)}}\right)dx$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = \int_{\pi/4}^{2\pi/3}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}}\right)dx$

$\int\left(\sqrt{1+\cos(x)}\right)dx = \int\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}}\right)dx$

$\int\left(\sqrt{1+\cos(x)}\right)dx = 2\sqrt{1-\cos(x)} + C$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = 2\sqrt{1-\cos(x)}\Big|_{\pi/4}^{\pi/3}$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = 2\sqrt{1-\cos(\pi/3)} - 2\sqrt{1-\cos(\pi/4)}$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = 2\sqrt{1-\frac{1}{2}} - 2\sqrt{1-\frac{\sqrt{2}}{2}}$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = \sqrt{2} - 2\sqrt{\frac{2-\sqrt{2}}{2}}$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = \sqrt{2} - (\sqrt{2})\sqrt{2-\sqrt{2}}$

$\int_{\pi/4}^{\pi/3}\left(\sqrt{1+\cos(x)}\right)dx = \sqrt{2} - \sqrt{4-2\sqrt{2}}$

Hence the answer is C

19. $y = x^2 + 6\arctan(4x)$

$\frac{dy}{dx} = 2x + 6\left[\frac{4}{1+(4x)^2}\right]$

$\frac{dy}{dx}\Big|_{x=1} = 2(1) + 6\left[\frac{4}{1+(4)^2}\right] = \frac{58}{17}$

Hence the answer is D

20. I is false. If a function is differentiable at $x = a$, then it is continuous at $x = a$. In this case, $x = a$ and $x = b$ are not necessarily differentiable and therefore there is insufficient information to say $f(x)$ is continuous at $x = a$ and $x = b$.

II is true. This is the Extreme Value Theorem.

III is true. A strictly increasing or strictly decreasing function is one-to-one

Hence the answer is B

21. $f'''(x) = 48x + 36$

$f''(x) = 24x^2 + 36x + C_1$

$f''(1) = 40 = 24 + 36 + C_1 \rightarrow \ C_1 = -20$

$f'(x) = 8x^3 + 18x^2 - 20x + C_2$

$f'(1) = 20 = 8 + 18 - 20 + C_2 \ \rightarrow \ C_2 = 14$

$f(x) = 2x^4 + 6x^3 - 10x + 14x + C_3$

$f(0) = 16 = C_3$

$f(x) = 2x^4 + 6x^3 - 10x^2 + 14x + 16$

$f(2) = 32 + 48 - 40 + 28 + 16 = 84$

Hence the answer is A

22. $\frac{d}{dx}\left(\int_a^u [f(t)]dt\right) = \frac{d}{du}\left(\int_a^u [f(t)]dt\right) \times \frac{du}{dx}$ where a is a constant

$\frac{d}{dx}\left(\int_a^u [f(t)]dt\right) = \frac{d}{du}\left(F(u) - F(a)\right) \times \frac{du}{dx}$

$\frac{d}{dx}\left(\int_a^u [f(t)]dt\right) = f(u) \times \frac{du}{dx}$

$f(x) = \frac{d}{dx}\int_1^{x-2}(t^2 + 6)dt$

$f(x) = (x-2)^2 + 6$

$f'(x) = 2(x-2)$

$f'(4) = 2(4-2) = 4$

Hence the answer is C

23. $\frac{dy}{dx} = 6y + 4xy$

$\frac{dy}{dx}\left(\frac{1}{y}\right) = 6 + 4x$

$\ln|y| = 6x + 2x^2 + C_1$

$y = C_2 e^{(6x+2x^2)}$

$y(1) = e^2 = C_2 e^{[6(1)+2(1)]}$

$C_2 = e^{-6}$

$y(4) = [e^{-6}]\left[e^{[6(4)+2(4)^2]}\right]$

$y(4) = e^{50}$

Hence the answer is B

24. $\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = \int_{\pi/4}^{5\pi/6}(\csc^2(x))(1+\cot^2(x))dx$

$\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = \int_{\pi/4}^{5\pi/6}[\csc^2(x)+\csc^2(x)\cot^2(x)]dx$

$\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = \left(-\cot(x)-\frac{1}{3}\cot^3(x)\right)\Big|_{\pi/4}^{5\pi/6}$

$\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = \left(-\cot\left(5\pi/6\right)-\frac{1}{3}\cot^3\left(5\pi/6\right)\right)-\left(-\cot\left(\pi/4\right)-\frac{1}{3}\cot^3\left(\pi/4\right)\right)$

$\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = \left(-(-\sqrt{3})-\frac{1}{3}(-\sqrt{3})^3\right)-\left(-(1)-\left(\frac{1}{3}\right)(1)\right)$

$\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = \left(\sqrt{3}-\frac{1}{3}(-3\sqrt{3})\right)-\left(-\frac{4}{3}\right)$

$\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = (\sqrt{3}+\sqrt{3})-\left(-\frac{4}{3}\right)$

$\int_{\pi/4}^{5\pi/6}(\csc^4(x))dx = \frac{4+6\sqrt{3}}{3}$

Hence the answer is A

25. $f^{(n)}(x)$ denotes the n$^{\text{th}}$ derivative of the function $f(x)$

$f(x) = (\sin(x))(\cos(x))$

$f(x) = \frac{1}{2}\sin(2x)$

$f'(x) = \cos(2x)$

$f^{(2)}(x) = -2\sin(2x)$

$f^{(3)}(x) = -4\cos(2x)$

$f^{(4)}(x) = 8\sin(2x)$

$f^{(5)}(x) = 16\cos(2x)$

$f^{(6)}(x) = -32\sin(2x)$

$f^{(7)}(x) = -64\cos(2x)$

$f^{(8)}(x) = 128\sin(2x)$

$f^{(9)}(x) = 256\cos(2x)$

$f^{(10)}(x) = -512\sin(2x)$

Hence the answer is B

26. THIS QUESTION WAS THROWN OUT because the Mean Value Theorem requires continuity at each endpoint of a given interval yet the given function has a vertical asymptote at the left endpoint x = 1.

~~$\int_a^b f(x)\,dx = \int_a^b \left[\sqrt{\frac{1}{4x-x^2-3}}\right] dx$~~

~~$\int_a^b f(x)\,dx = \int_a^b \left[\sqrt{\frac{1}{1-(x-2)^2}}\right] dx$~~

~~$\int_a^b f(x)\,dx = Arcsin(x-2)+C$~~

~~$\int_1^{\frac{3}{2}} f(x)\,dx = Arcsin(1-2) - Arcsin\left(\frac{3}{2}-2\right)$~~

~~$\int_1^{\frac{3}{2}} f(x)\,dx = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$~~

~~$f(c) = \left(\frac{\frac{\pi}{3}}{1-\frac{3}{2}}\right) = \frac{2\pi}{3}$~~

~~$f(c) = \sqrt{\frac{1}{1-(c-2)^2}}$~~

~~$\frac{1}{1-(c-2)^2} = \frac{4\pi^2}{9}$~~

~~$1-(c-2)^2 = \frac{9}{4\pi^2}$~~

~~$(c-2)^2 = \frac{4\pi^2-9}{4\pi^2}$~~

~~$c = 2 \pm \sqrt{\frac{4\pi^2-9}{4\pi^2}} = 2 \pm \frac{\sqrt{4\pi^2-9}}{2\pi}$~~

~~**Hence the answer is D**~~

27. $f(x) = x^2 - x - 6$

$f'(x) = 2x - 1$

$y = f(x_n) + (x_{n+1} - x_n)f'(x_n)$ → isolate for x_{n+1} when $y = 0$

$x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \left(\frac{-4}{3}\right) = \frac{10}{3}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{10}{3} - \left(\frac{16}{51}\right) = \frac{154}{51}$

Hence the answer is A

28. $f(\theta) = 3^{\tan(\theta)}$

$\ln\big(f(\theta)\big) = \tan(\theta)\ln(3)$

$\frac{f\prime(\theta)}{f(\theta)} = [\sec^2(\theta)]\ln(3)$

$f'(\theta) = 3^{\tan(\theta)}[(\sec^2(\theta))\ln(3)]$

$f'\left(-\frac{\pi}{4}\right) = 3^{\tan\left(-\frac{\pi}{4}\right)}\left[\left(\sec^2\left(-\frac{\pi}{4}\right)\right)\ln(3)\right]$

$f'\left(-\frac{\pi}{4}\right) = 3^{-1}\left[\left(\sqrt{2}\right)^2\ln(3)\right]$

$f'\left(-\frac{\pi}{4}\right) = \frac{1}{3}[2\ln(3)]$

$f'\left(-\frac{\pi}{4}\right) = \frac{2}{3}\ln(3) = \ln\left(\sqrt[3]{9}\right)$

Hence the answer is B

29. $u = x \rightarrow du = dx$

$dv = e^{2x}dx \rightarrow v = \frac{1}{2}e^{2x}$

$\int(xe^{2x})dx = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx$

$\int(xe^{2x})dx = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$

$\int_0^2(xe^{2x})dx = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}\Big|_0^2$

$\int_0^2(xe^{2x})dx = \left[e^4 - \frac{1}{4}e^4\right] - \left[0 - \frac{1}{4}\right]$

$\int_0^2(xe^{2x})dx = \frac{1+3e^4}{4}$

Hence the answer is C

30. $\lim_{x\to 2}\left(\frac{x^3+x^2-4x-4}{x-2}\right) = \lim_{x\to 2}\left(\frac{x^3-4x+x^2-4}{x-2}\right)$

$\lim_{x\to 2}\left(\frac{x^3+x^2-4x-4}{x-2}\right) = \lim_{x\to 2}\left(\frac{x(x^2-4)+(x^2-4)}{x-2}\right)$

$\lim_{x\to 2}\left(\frac{x^3+x^2-4x-4}{x-2}\right) = \lim_{x\to 2}\left(\frac{(x^2-4)(x+1)}{x-2}\right)$

$\lim_{x\to 2}\left(\frac{x^3+x^2-4x-4}{x-2}\right) = \lim_{x\to 2}\big((x+2)(x+1)\big)$

$\lim_{x\to 2}\left(\frac{x^3+x^2-4x-4}{x-2}\right) = (2+2)(2+1) = 12$

Hence the answer is D