AHS-PB Invitational February 2022 **Pre-Calculus Individual Test**

The acronym NOTA stands for "None of the above answers is correct." Good luck!

1.) Given an arithmetic sequence where $a_5 = 12$ and $a_9 = 28$, find a_{14}. Note, a_1 is denoted as the first term.

A) 44 B) 48 C) 52 D) 56 E) NOTA

2.) The area that is enclosed by the triangle on the xy-plane with vertices $(7,1)$, (a,b), and $(-2,-4)$ can be written in the form $\frac{|xa+yb+z|}{2}$ where there is no nontrivial common divisor between x, y and z and where $x < 0$. Find the units digit of the value of $3|x| + 2|y| - |z|$.

A) 3 B) 5 C) 7 D) 9 E) NOTA

3.) The slope of the line that contains the set of all points where the area that is enclosed by the triangle on the xy-plane with vertices $(7,1)$, (a,b), and $(-2,-4)$ is 0 can be written in the form $\frac{c}{d}$ where $c < d$. Find the units digit of the expression $c + 2d$.

A) 1 B) 3 C) 4 D) 5 E) NOTA

4.) Find the sum of the following infinite sequence: $9,4,\frac{16}{9},\frac{64}{81},\frac{256}{729}$ ...

A) $\frac{81}{4}$ B) $\frac{81}{5}$ C) $\frac{101}{4}$ D) $\frac{101}{5}$ E) NOTA

Use the following information for problems 5-7. At DBHS Mr. Snow has a total of 64 math students. Every one of Mr. Snow's math students takes zero, one, or more of the following foreign language classes: Spanish, French, and/or Japanese. 39 students of Mr. Snow's students take Spanish, 25 take French, 13 take Japanese. 28 take Spanish only, 16 take French only, and 7 take Japanese only. Only 2 students take all 3 foreign languages.

5.) If Mr. Snow randomly selects one of his math students, what is the probability that they take 2 or more foreign language classes?

A) $\frac{1}{64}$ B) $\frac{3}{16}$ C) $\frac{5}{16}$ D) $\frac{3}{8}$ E) NOTA

6.) The probability that Mr. Snow chooses a student who takes Spanish, given that he randomly chooses from the pool of students who take only one of these foreign language classes can be written in the form $\frac{c}{d}$ where $c < d$ and that $\frac{c}{d}$ is in reduced fraction form. Find the units digit of the expression $2c - d$.

A) 0 B) 2 C) 5 D) 9 E) NOTA

7.) The probability that Mr. Snow chooses a math student who takes French, given that he randomly chooses from the pool of students who do not take Spanish can be written in the form $\frac{m}{n}$ where $m < n$ and that $\frac{m}{n}$ is in reduced fraction form. Find the units digit of the expression $2n - m$.

A) 0 B) 3 C) 5 D) 9 E) NOTA

Use the following conic section for problems 8-10.

$$x^2 + 2y^2 - 4x + 12y = 14$$

8.) Find the length of the major axis of the conic section.

A) 3 B) 6 C) 9 D) 12 E) NOTA

9.) Find the distance between the origin and the center of the conic section.

A) $\sqrt{2}$ B) $\sqrt{5}$ C) $\sqrt{13}$ D) $\sqrt{17}$ E) NOTA

10.) Find the value of the focal length of the conic section.

A) $\frac{\sqrt{2}}{4}$ B) $\frac{3\sqrt{2}}{2}$ C) $\sqrt{2}$ D) $3\sqrt{2}$ E) NOTA

11.) If $f(x)$ is an even function, $g(x)$ is an odd function, and $h(x) = f\big(g(x)\big)$ which of the following statements must be true?

I. $h(x)$ is an even function.

II. $f(h(x))$ is an odd function.

III. There is not enough information to determine if $h(x)$ is either odd or even.

IV. $f(h(x))$ and $g(h(x))$ both are not odd, and both are not even.

A) I only B) I and II only C) III only D) III and IV only E) NOTA

12.) Find the vertex of the following parabola:

$x = y^2 - 6y + 9.$

A) (0, -3) B) (-3, 0) C) (0,3) D) (3,0) E) NOTA

13.) Find the number of intersections between the functions $y = x^2$ and $y = \sin^2 x + \cos^2 x.$

A) 0 B) 1 C) 2 D) 4 E) NOTA

14.) Solve the following equation for x: $logx^2 - log2x = 0$

A) 2 B) 3 C) 4 D) 5 E) NOTA

15.) THIS QUESTION WAS THROWN OUT!

~~When the equation $r = 2\,sin\,\theta - \theta$ is converted to rectangular form it can be written precisely in the form $ax^2 + by^2 + cx + dy + \frac{fx^2}{\sqrt{gx^2+hy^2}} = 0$ where there is no non-trivial common divisor between the integers a, b, c, d, f, g and h and where $a > 0$. Find the value of the expression $a + 2b + 3c + 4d + 5f + 6g + 7h$.~~

~~A) 13 B) 16 C) 25 D) 28 E) NOTA~~

16.) Evaluate the period of the equation $y = 3sin\,(2x - \pi) + 2$

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) 2π E) NOTA

17.) Evaluate the phase shift of the equation $y = 3sin\,(2x - \pi) + 2$.

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) 2π E) NOTA

18.) Find the area of the sector whose central angle is $\frac{2\pi}{3}$ and whose radius is $\sqrt{2}$.

A) $\frac{2\pi}{3}$ B) $\frac{2\pi\sqrt{2}}{3}$ C) $\frac{4\pi}{3}$ D) $\frac{4\pi\sqrt{2}}{3}$ E) NOTA

19.) Find the number of the distinct permutations of the letters in the word LEVELS.

A) 60 B) 90 C) 120 D) 180 E) NOTA

20.) Find the area of a triangle with side lengths 4 and 6 with an included angle of $\frac{5\pi}{6}$.

A) 6 B) 8 C) 10 D) 12 E) NOTA

21.) Let the graph of $f(x) = \frac{x^3-x}{x^3+3x^2+2x}$ have 2 removable discontinuities (holes) at $x = a$ and $x = b$ where $a > b$ and it also has a vertical asymptote at $x = c$ and a horizontal asymptote at $y = d$. Find the value of the expression $|3a + 2b + c - d|$.

A) 4 B) 5 C) 7 D) 9 E) NOTA

22.) Triangle ROB has RO = 6, OB = 12, and BR = 13. The exact value of the cosine of angle R can be written in the form $\left(\frac{a}{b}\right)$ where $a < b$. Find the units digit of $a + b$.

A) 4 B) 5 C) 7 D) 9 E) NOTA

Use the following information for problems 23-24. $P(A) = .3, P(B) = .5, P(A \cap B) = .1$

Note, the notation of A^c means the complement of the event A.

23.) Find the value of $P(A \cup B)^c$.

A) .1 B) .2 C) .3 D) .4 E) NOTA

24.) Find the value of $P(A^c \cap B) - 2P(A \cap B^c)$.

A) .1 B) .2 C) .3 D) .4 E) NOTA

25.) Evaluate $\sum_{i=6}^{16} i$

A) 96 B) 121 C) 136 D) 145 E) NOTA

26.) Evaluate $tan\ 2x$ given that $tan\ x = \frac{1}{2}$.

A) $\frac{4}{3}$ B) 1 C) $\frac{2}{3}$ D) $\frac{1}{3}$ E) NOTA

27.) Find the sum of the digits of the following: $45^{\circ}12'900^{"}$ into degrees only, using decimal notation. Note, the notation of the given value is written in degrees, minutes, seconds.

A) 9 B) 12 C) 15 D) 18 E) NOTA

28.) The angle between the vectors $< 5, -1 >$ and $< 4, -4 >$ can be written in the form $cos^{-1}(\frac{a\sqrt{b}}{c})$ when fully simplified in reduced radical form. Find the value of $2a + b + 2c$.

A) 15 B) 25 C) 36 D) 45 E) NOTA

29.) Find the value of a such that the two vectors $< 2, -1 >$ and $< a, 6 >$ are perpendicular.

A) 1 B) 2 C) 3 D) 4 E) NOTA

30.) Evaluate the dot product $< 2, 4 > \cdot < 3, -2 >$.

A) -6 B) -2 C) 2 D) 6 E) NOTA