The acronym NOTA denotes that "none of the above answers is correct." Trigonometric Arc functions are defined over the traditional restricted ranges. Let $\text{cis}(\theta) = \cos(\theta) + i\sin(\theta)$. The value 0^0 is considered undefined. All answers are rationalized, simplified, and exact unless otherwise stated. Have Fun!

1. Which of the following represents the distance between the polar coordinate points $\left(3\sqrt{3}, \frac{\pi}{3}\right)$ and $\left(\sqrt{5}, \frac{5\pi}{6}\right)$?

A. $2\sqrt{2}$ B. $4\sqrt{2}$ C. $\sqrt{235}$ D. $\sqrt{305}$ E. NOTA

2. The area of a triangle is 56 and is formed with vertices $(3,3)$, (x, y), and $(6,2)$. Given all possible values for *x* and *y* are positive integers, which of the following represents the number of possible distinct values for *y*?

A. 41 B. 42 C. 75 D. 123 E. NOTA

3. Find the number of solutions for *x*: $x^3 = -8$

A. 6 B. 3 C. 2 D. 1 E. NOTA

For questions 4-5: consider the conic defined by the equation $r = \frac{6}{4-3\sin(\theta)}$

4. Which of the following represents the shape of the conic?

A. Parabola B. Hyperbola C. Ellipse D. Circle E. NOTA

5. Which of the following in an equation of a directrix of the above conic?

A. $x = -2$ B. $y = -2$ C. $x = 2$ D. $y = 2$ E. NOTA

6. Compute: $\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)$

A. $\frac{\sqrt{6}-\sqrt{2}}{4}$ B. $\frac{\sqrt{2-\sqrt{3}}}{2}$ C. $\frac{2-\sqrt{3}}{4}$ D. $\frac{2+\sqrt{3}}{4}$ E. NOTA

7. Find the length of the shortest altitude in a triangle with side lengths 9, 12, and 15

A. $\frac{36}{5}$ B. 9 C. 12 D. $\frac{72}{5}$ E. NOTA

8. Solve for the sum of the distinct rational real values of x: $(x^2 - 3x - 17)^{(x^2+x-6)} = 1$

A. 8 B. 7 C. 6 D. 5 E. NOTA

9. If $z_A = 4 + 8i$, what is the value of $\left|\frac{1}{z_A}\right|$?

A. $\frac{\sqrt{5}}{32}$ B. $\frac{\sqrt{5}}{20}$ C. $\frac{2\sqrt{5}}{35}$ D. $\frac{\sqrt{5}}{12}$ E. NOTA

10. Which of the following represents the acute angle formed by intersection of lines $L_1 = \begin{cases} x = 4 + 8t \\ y = 1 - 4t \end{cases}$ and $L_2 = \begin{cases} x = 1 + 6t, \\ y = 3t - 4 \end{cases}$?

A. $\arccos(-1)$ B. $\arccos\left(-\frac{7}{30}\right)$ C. $\arccos\left(\frac{3}{5}\right)$ D. $\arccos\left(\frac{18}{25}\right)$ E. NOTA

11. Given $\sin(\theta) - \cos(\theta) = \frac{1}{2}$ and θ is on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, what is the value of $\cos(2\theta)$?

A. $^3/_4$ B. $^{\sqrt{7}}/_4$ C. $^{\sqrt{7}}/_{16}$ D. $^1/_2$ E. NOTA

12. Which of the following is coterminal with $3120°$?

A. $120°$ B. $140°$ C. $240°$ D. $340°$ E. NOTA

13. Compute:

$$\sum_{n=1}^{8} \begin{vmatrix} \sin\left(\frac{n\pi}{4}\right) & \cos\left(\frac{n\pi}{4}\right) \\ \sin\left(-\frac{n\pi}{4}\right) & \cos\left(-\frac{n\pi}{4}\right) \end{vmatrix}$$

A. 0 B. $2\sqrt{2}$ C. 4 D. $4\sqrt{2}$ E. NOTA

14. Find the period of the function $f(x) = 2 - 2\cos^2(\theta)$

A. $\frac{\pi}{8}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{2}$ D. π E. NOTA

15. Which of the following is equivalent to the inverse of the matrix $\begin{bmatrix} 28 & -16 \\ 8 & -4 \end{bmatrix}$?

A. $\begin{bmatrix} -7/4 & -1 \\ 1/2 & 1/4 \end{bmatrix}$ B. $\begin{bmatrix} -1/4 & 1 \\ -1/2 & 7/4 \end{bmatrix}$ C. $\begin{bmatrix} -1/4 & -1 \\ 1/2 & 7/4 \end{bmatrix}$ D. $\begin{bmatrix} 7/4 & 1 \\ -1/2 & -1/4 \end{bmatrix}$ E. NOTA

16. Evaluate: $\cos\left(\operatorname{arccot}\left(-\frac{\sqrt{3}}{3}\right)\right)$

A. $\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{3}}{2}$ C. $\frac{1}{2}$ D. $-\frac{1}{2}$ E. NOTA

17. How many petals are in the graph of $r = 3\cos(2020\theta)$?

A. 4040 B. 3030 C. 2020 D. 1010 E. NOTA

18. Express in Cartesian form: $\left(\sqrt{6} - i\sqrt{2}\right)^8$

A. $-1024 + 1024\sqrt{3}i$ C. $2048\sqrt{3} - 2048i$ E. NOTA

B. $-4096 + 4096\sqrt{3}i$ D. $-2048 + 2048\sqrt{3}i$

19. What is the maximum possible value of the function $f(x) = 2\sin(x) + \cos(2x)$?

A. 3 B. $\frac{3}{2}$ C. $\frac{11}{8}$ D. -3 E. NOTA

20. What is the area of the region formed by the graph $|z + 3| = 2|z - 2i|$, where $z = x + yi$, on the complex plane?

A. $\frac{52}{9}\pi$ B. $\frac{56}{9}\pi$ C. 14π D. 60π E. NOTA

21. The function $g(x) = A\cos\left(Bx + \frac{\pi}{6}\right) + D$ has a period of $\frac{\pi}{8}$ and $g\left(\frac{\pi}{3}\right) = 14$. If the vertical distance from the **top of the graph** to the **bottom of the graph** is 16 and *B* is positive, then compute $|AD|$.

A. 32 B. 48 C. 112 D. 224 E. NOTA

22. Evaluate $\log_{49}(343) + \log_{49}(7) - \log_2 512$

A. -9 B. -7 C. -6 D. -1 E. NOTA

For questions $23 - 24$, $z_1 = 2 - 4i$ and $z_2 = 4 + 2i$

23. The argument of a complex number is the angle made by the vector of the complex number and the real axis. Which of the following is gives an argument of $z_1 z_2$?

A. $\arctan\left(-\frac{3}{4}\right)$ B. $\arctan\left(-\frac{5}{4}\right)$ C. $\arctan\left(\frac{2}{3}\right)$ D. $\arctan\left(\frac{5}{4}\right)$ E. NOTA

24. Express in polar form: $\frac{z_2}{z_1}$

A. $cis\left(\frac{\pi}{4}\right)$ B. $cis\left(\frac{\pi}{2}\right)$ C. $cis\left(\frac{3\pi}{2}\right)$ D. $cis(0)$ E. NOTA

25. Solve for the possible values of x: $|x+3|-|x-1|>1$

A. $\left(-\frac{1}{2},3\right)$ B. $(-3,\infty)$ C. $\left(-\frac{1}{2},\infty\right)$ D. $\left(-\infty,-\frac{3}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$ E. NOTA

26. Which of the following vector(s) is/are perpendicular to the plane $5x-4z=6y+11$?

I. $\langle 6,3,3\rangle$ II. $\langle 4,8,-7\rangle$ III. $\langle -2,5,-4\rangle$

A. I and II B. II and III C. I only D. I, II, and III E. NOTA

27. Compute: $\frac{2^6-3^6}{2^3+3^3}$

A. 35 B. -1 C. -5 D. -19 E. NOTA

28. Find the sum of the non-real roots: $f(x)=x^3-3x^2+5x-15$

A. 0 B. 3 C. $10i$ D. 13 E. NOTA

29. Classify the conic: $f(x,y)=5x^2+4xy-3y^2+6y+23$

A. Parabola B. Hyperbola C. Ellipse D. Circle E. NOTA

30. Determine $(x-y)$ if $\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}\begin{bmatrix}x & 2\\ y & 5\end{bmatrix}=\begin{bmatrix}11 & 1\\ 26 & 24\end{bmatrix}$

A. 9 B. $^9/_7$ C. 1 D. -3 E. NOTA