February Regional Geometry Team Question #1.

Write “True” or “False” for each of the following statements:

A) All squares are rhombuses.

B) All squares are trapezoids.

C) All squares are kites.

D) All rectangles are rhombuses.

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February Regional **Geometry Team** **Question #2.**

In parallelogram $ABCD$, $m\angle C = 4x$, $m\angle D = x - 2y$, and $m\angle A = 160^\circ$

A) What is the value of y?
B) What is the value of x?
C) What is $m\angle B$?
D) What is $m\angle C$?

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February Regional **Geometry Team** **Question #3.**

Consider each of the following sets of numbers:

i) 2,3,4

ii) 10,10,20

iii) 8,15,17

iv) 7,9,13

v) 11,12,15

vi) 5,12,13

vii) $\frac{1}{4}, \frac{1}{2}, \frac{5}{8}$

viii) $\sqrt{8}, \sqrt{10}, \sqrt{18}$

A) How many of the above can be the side lengths of an acute triangle?
B) How many of the above can be the side lengths of an obtuse triangle?
C) How many of the above can be the side lengths of a right triangle?
D) How many of the above cannot be the side lengths of a triangle.

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February Regional **Geometry Team** **Question #4.**

A) For right triangle ABC with $m\angle B = 90°$, what is $(tan\, A)(tan\, C)$?
B) What is the perimeter of a rhombus with diagonals 10 and 24?
C) What is the area of the incircle of a triangle with side lengths 9,40, 41?
D) What is the perimeter of an isosceles trapezoid with area 180 and bases 10 and 20?

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February Regional Geometry Team Question #5.

In regular hexagon *ABCDEF* with side length 6:

A) What's the measure of angle *BEC*?
B) What's the measure of angle *EBA?*
C) What is the area of triangle *BEC?*
D) What is the area of triangle *CDE?*

February Regional Geometry Team Question #5.

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February Regional **Geometry Team** **Question #6.**

In a circle with radius 12:

A) What's the area of a 45-degree sector?
B) What's the "perimeter" of a 45-degree sector cut out of the circle?
C) If a regular hexagon is inscribed in the circle, what is the ratio of the circumference of the circle to the perimeter of the hexagon?
D) If a regular hexagon is inscribed in the circle, what is the ratio of the area of the circle to the area of the hexagon?

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February Regional **Geometry Team** **Question #7.**

A) Three points *X, Y,* and *Z* lie on the plane. $X = (2,4)$, and if the midpoint of segment $\overline{XY}$ is $(-1,2)$ and the midpoint of $\overline{XZ}$ is $(4,1)$, then find the length of $\overline{YZ}$.
B) Using the information in part A), what is the centroid of triangle *XYZ* ?
C) How many sides does a regular polygon with 54 diagonals have?
D) In a regular polygon with 170 diagonals, what is the measure of each exterior angle?

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February Regional **Geometry Team** **Question #8.**

A) A circle with center *O* has minor arc *AB,* on which lies point *N*. If $m\angle AOB = 72°$, then what is $m\angle ANB$?

B) Square *ABCD* has side length 6. There is a circle centered at C, and points *B* and D lie on the circle. Find the area of the portion of the circle inside the square.

C) For two circles *A* and *B*, the diameter of circle *B* is 10 times bigger than the diameter of circle *A.* If circle *B* has an area of 24, find the area of circle *A*.

D) Suppose *ABC* is an equilateral triangle with side length 8. *D* and *E* are the midpoints of side $\overline{AB}$ and side $\overline{AC}$, respectively. What's the height of trapezoid *BDEC*?

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February Regional **Geometry Team** **Question #9.**

Find the area of each of the following:

A) An equilateral triangle with altitude 6.
B) The region between the x-axis, y-axis, and $2x + y = 4$.
C) A rhombus with perimeter 80 and one diagonal has length 24
D) For rhombus *AMRU*, suppose $RU = AR = 2$, what is the area of the rhombus?

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February Regional **Geometry Team** **Question #10.**

A) What is the measure of one interior angle in a regular 12-gon?
B) What is the sum of the supplements of each exterior angle in a regular decagon?
C) What is the sum of the complements of each exterior angle in a regular 15-gon?
D) What is the measure of one interior angle in a regular heptagon?

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February Regional **Geometry Team** **Question #11.**

A) What is the area of the circle circumscribing an equilateral triangle of area $12\sqrt{3}$?
B) What is the area of the circle circumscribing a square of area 12?
C) What is the area of the circle circumscribing a hexagon of area $12\sqrt{3}$?
D) What's the area of a square if the circle inscribed in the square has area 12?

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February Regional Geometry Team Question #12.

Triangle *ABC* and triangle *DEF* are similar. *AB* = 7, *DE* = 42, *DF* = 12, and *EF = 48*.

A) What's the length of *AC*?
B) What's the area of triangle *ABC*?
C) What is the area of triangle *DEF*?
D) Is triangle *ABC* acute, obtuse, or right? (Assign 1 = acute, 2 = obtuse, or 3 = right)

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February Regional Geometry Team Question #13.

For parts A-C, Triangle *ABC* is equilateral and has side length 8.

A) What is the length of each side of an equilateral triangle with a fourth of the area of triangle *ABC*?
B) What is the area of an equilateral triangle with each side three times the length of each side of triangle *ABC*?
C) What is the distance between the centroid and orthocenter in triangle *ABC?*
D) In right triangle *DEF* with $m\angle E = 90°$, if $\sin F = 1/6$, find $\sin D$.

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February Regional **Geometry Team** **Question #14.**

A) In a triangle with lengths 65,156,169, what is the length of the shortest altitude?

B) Find the arithmetic mean of $3\sqrt{3}$ and $5\sqrt{3}$.

C) Find the geometric mean of $3\sqrt{3}, 6\sqrt{3}$, and 4.

D) Find the harmonic mean of $3\sqrt{3}$ and $5\sqrt{3}$.

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February Regional Geometry Team Question #15.

For part A and B: A triangle is defined with vertices $A(1,2), B(3,5)$, and $C(2,7)$?

A) What is the area of the triangle?
B) What is the sum of the squares of each side of the triangle?

C) Consider the line $2x - y = 3$. Find the sum of possible values for g so that the distance between $(6, g)$ and the line is 5.
D) Triangle *KIT* is on the coordinate plane, with point $T = (5,6)$, and points *K* and *I* are on the line $x = 1$. If $m\angle KTI = 60°$ and $m\angle IKT = 30°$, what's the length of the longest side of the triangle?

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