Answer Key:

1. B
2. B
3. B
4. E
5. D

6. C
7. C
8. B
9. A
10. C

11. B
12. B
13. B
14. B
15. B

16. D
17. D
18. B
19. E
20. C

21. D
22. A
23. C
24. B
25. D

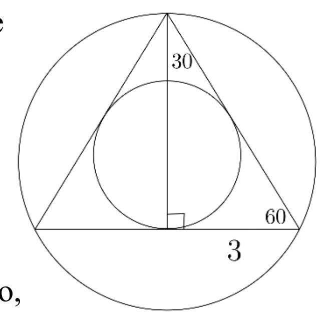
26. A
27. C
28. B
29. C
30. C

Solutions:

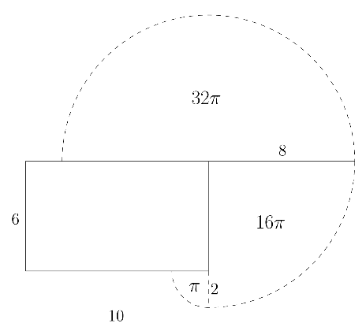
1. We can find the area of an equilateral triangle using the formula $\frac{s^2\sqrt{3}}{4}$, where *s* is the side length of the triangle. In this case we plug in 6 to obtain **(B)** $9\sqrt{3}$.

2. Since this is an isosceles triangle, two sides must have the same value. This leads to two possibilities, a triangle with sides 5,5,12, or a triangle with sides 5,12,12. However, the first doesn't make a triangle (since $5+5 \ngtr 12$), so the third side must be **(B)** 12.

3. We'll first find the altitude of this triangle, and since it is equilateral, the altitude splits the triangle into two 30-60-90 triangles. Then, the length of the base of one of these triangles is 3, so the altitude has length $3\sqrt{3}$. The length of the radius of the circumcircle is 2/3 of the length of the altitude of the triangle, and the radius of the incircle is 1/3 of the length of the altitude of the triangle. Thus, the radius of the circumcircle is $2\sqrt{3}$ and the radius of the incircle is $\sqrt{3}$. Then, to find the difference in area we would do $\pi R^2 - \pi r^2 = \pi(R-r)(R+r) = \pi(\sqrt{3})(3\sqrt{3}) = 9\pi$. So, the answer is **(B)** 9π.

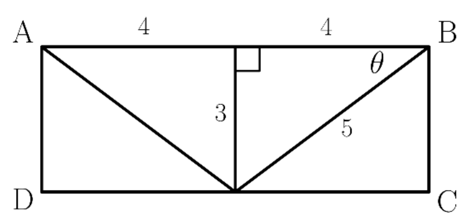


4. It is the sum of the following areas in the figure, which we found by either taking a half or a quarter of the area of a circle (πr^2). This is 49π, so the answer is **(E)** NOTA.



5. This is using the formula from question 1, setting it equal to $2\sqrt{3}$ to find that each side has length $2\sqrt{2}$**.** Hence the answer is **(D)** $2\sqrt{2}$.

6. Sine is equal to opposite over hypotenuse, and then since this is a right triangle, we know the other leg of the triangle is 4. We see that for the rectangle, the base is 8 and the height is 3, so the area is $bh = 8 \times 3 = 24$. Hence the answer is **(C)** 24**.**



7. They share the same base, so let's call it *b*. Then the ratios of the areas is $\frac{1/2b8}{1/2b4} = 2$. This is choice **(C)**.

8. This can be solved using rules of 30-60-90 triangles and 45-45-90 triangles to see that $BD = 2\sqrt{3}$ and $DC = 6$. Then we can find the area using that the height is 6 and base is $6 + 2\sqrt{3}$, which is **(B)** $18 + 6\sqrt{3}$.

9. Since this is a triangle inscribed in a semicircle, the inscribed angle C has a measure of 90 degrees. Since angle *CBA* is an inscribed angle corresponding to arc *AC*, it has a measure of 60 degrees. This means triangle *ABC* is a 30-60-90 triangle and consequently *BC* is half of *AB*, making it **(A)** 5.

10. The largest exterior angle of a right isosceles triangle is 135 degrees. The supplement is $180 - 135 = 45$ degrees, and then the complement of that is $90 - 45 = 45$ degrees **(C)**.

11. By the triangle inequality, the third side must be greater than 3 and less than 5. Since the sides of the triangle are all integers, it only can be 4, which would be both the largest possible and smallest possible side length. Hence the areas would be the same, leaving a difference of **(B)** 0.

12. We can see that $AD = AB + BC + CD$, and using the given information, $24 = 3CD + \frac{3CD + CD}{2} + CD = 6CD$. This implies $CD = 4$, and hence $AB = 12$. Since *BC* is the average of these two values, $BC = 8$ (**B**).

13. This is finding the centroid of a triangle. $(\frac{1-3-7}{3}, \frac{2+3+7}{3}) = (-3, 4)$ **(B)**.

14. This is equivalent to finding the area of a square inscribed in a circle of diameter 14. In this case the diagonal of the square would be 14, and since this is a 45-45-90 triangle each side length of the square is $7\sqrt{2}$, so the area is $49 \times 2 = 98$ **(B)** feet squared.

15. A 3×4 inch picture has a diagonal of 5 inches. Since $75/5 = 15$, we scale the length and width by a factor of 15. Then scaling 3×4 inches by 15 we obtain 45×60 **(B)** inches.

16. The area of this square is 36. Since all lengths are doubled, the areas would be quadrupled. By this principle, the area under a magnifying glass would be $36 \times 4 = 144$ **(D)**.

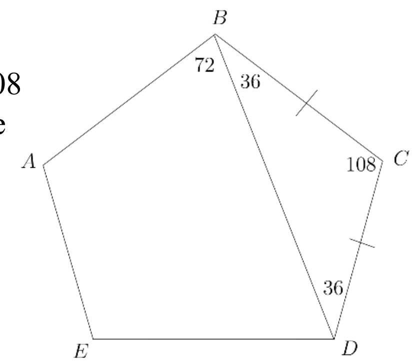
17. If the interior angle is 175 degrees, then the exterior angle is 5 degrees, which is true for every angle of the polygon since it is regular. $360/5 = 72$, so, we have **(D)** 72 sides.

18. We complete the square and put the constant term on the other side of the equal sign to obtain the equation $(x-2)^2 + (y+3)^2 - 4 - 9 - 23 = 0$, which then becomes $(x-2)^2 + (y+3)^2 = 36$. This means the radius is 6, so the circumference becomes $2\pi r = 12\pi$ **(B).**

19. First consider a clock, which has a mark for each minute. There are 12 hours, with 5 marks between each hour, so 60 marks on the whole clock. Since a circle is 360 degrees, this means the space between two marks is 6 degrees. Let's call the mark at the 25-minute mark the prime mark. The minute hand is at the 26th minute so 6 degrees past the prime mark. There are 30 degrees between the 25-minute mark and the 30-minute mark, so we can find exactly how many degrees past the 25-minute mark this is by doing

$26/60\times30$, which is 13 degrees. Since the minute hand is 6 degrees past the prime mark and the hour hand is 13 degrees past the prime mark, the hour and minute hands are 7 degrees apart. This is none of the answer choices, so the answer is **(E) NOTA.**

20. Drawing a pentagon, we see that in triangle BCD, angle C has measure of 108 degrees (each interior angle of a regular pentagon), and since this is isosceles, the two other angles must be equal. This means we can find the measure of angle CBD to be $\frac{180-108}{2}=36$. Since the measure of angle B is 108, to find the measure of angle ABD we can see $108-36=72$ **(C)**.



21. Since angle ACB is an inscribed angle, arc AB has a measure of 40 degrees, which since the measure of arc BAC is 190, the measure of arc AC is 150. Angle B is the inscribed angle corresponding to arc AC, so the measure of angle B is 75 **(D)**.

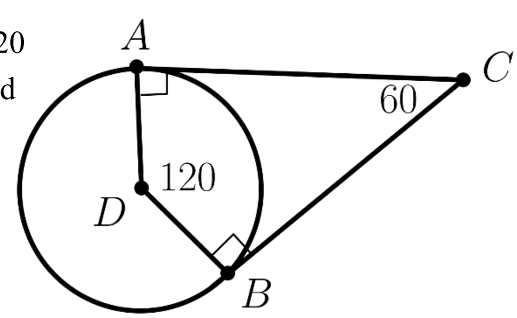
22. We must calculate the area of how much pizza each person eats. Bailey eats two squares of side length 8, so this area is $2\times8^2=128in^2$. For Fiona, it is $2(\pi(6)^2-8^2)=72\pi-128$. For Hanna, it is $2(\pi8^2-\pi6^2)=56\pi$. So, 56π, 128, and $72\pi-128$ are approximately 168, 128, 88 if you multiply by 3 as a rough estimate for π. So… **(A)** Hanna, Bailey, Fiona

23. For this we use Euler's formula for polyhedra, F + V = E + 2, where F is the number of faces, V the number of vertices, and E the number of edges. In this case it has 12 edges and 6 vertices, and using the formula, we see this implies there are 8 faces **(C)**.

24. By the angle bisector theorem, $\frac{BD}{8}=\frac{16-BD}{10}$. This becomes $10BD=128-8BD$, and solving for *BD*, we obtain $\frac{64}{9}$ **(B)**.

25. Since they are similar triangles, they are proportional, and for this case we can say $\frac{BA}{CA}=\frac{FD}{ED}$, which becomes $\frac{6}{8}=\frac{FD}{12}$, and solving for *FD* results in 9 **(D)**. This is the same as *DF*.

26. Let D denote the center of circle O. The measure of angle ADB is 120 since it is the central angle corresponding to minor arc AB. Since $\overline{CB}$ and $\overline{AC}$ are tangent to the circle and if we observe quadrilateral ADBC, then the measure of angles A and B is 90 each. Since the sum of angles in a quadrilateral is 360, the measure of angle C is $360-(90+90+120)=60$ **(A)**.



27. This can be solved by the handshake formula, which is $\frac{n(n-1)}{2}$, where n is the number of people. A more general approach: This problem is equivalent to finding the number of subsets with 2 elements from a set of 10 elements. That is a combination, in particular $\binom{10}{2}$, which is 45 **(C)**.

28. From a secant theorem we have the relation $(EA)(EB) = (EC)(ED)$, which becomes $(2EC)(2EC+7) = (EC)(EC+56) \Rightarrow 4EC^2 + 14EC = (EC^2 + 56EC) \Rightarrow 3EC^2 - 42EC = 0$, which means $EC = 14$ **(B).**

29. Carl Friedrich Gauss constructed the heptadecagon for the first time, in 1796 at the age of 19. Hence the answer is **(C).**

30. Since we can use parts of cupcakes, this is the area of the rectangle divided by the area of the base of a cupcake, which is a hexagon. The area of the rectangle is $20 \times 30 = 600$ and the area of the base of a cupcake is $6 \times \frac{1^2\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$. To see how many cupcakes will fit, we do $600/(\frac{3\sqrt{3}}{2})$, which becomes $\frac{400\sqrt{3}}{3}$ **(C).**