February Regional Calculus Team Question #1.

Legosi and Haru are growing flowers together. They both initially plant their flowers at $t = 0$. The height (in inches) of Legosi's flower t weeks after planting can be modeled by $L(t) = 4t - t^2$. The height (in inches) of Haru's flower t weeks after planting can be modeled by $H(t) = 6(1 - 2^{-t})$.

A. Let A denote the rate of growth of Legosi's flower (in inches per week) at $t = 1$.
B. Let B denote the height (in inches) that Haru's flower approaches as t grows large.
C. Let C denote the maximum height of Legosi's flower (in inches).
D. Let D denote the unique value of T such that the average rate of growth of Haru's flower over $0 \leq t \leq T$ is 3 inches per week.

Compute $A + B + C + D$.

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Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #2.**

The function $\lfloor x \rfloor$ returns the greatest integer less than or equal to x.

A. Let $A = \lim_{x\to\infty} \frac{20+22x-24x^2}{2x^2+1}$
B. Let $B = \lim_{x\to 2} \frac{x^2-22x+40}{x^2-10x+16}$
C. Let $C = \lim_{x\to 0^+} (\lfloor 20\sin(x) \rfloor + \lfloor 22\cos(x) \rfloor)$
D. Let $D = \lim_{x\to 6^-} (\lfloor x^2 \rfloor - \lfloor x \rfloor^2)$

Compute $A + B + C + D$.

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Compute $A + B + C + D$.

February Regional — Calculus Team — Question #3.

The function $\lfloor x \rfloor$ returns the greatest integer less than or equal to x.

A. Let $A = \frac{d^3}{dx^3}[x^5 e^x]_{x=0}$
B. Let $B = \frac{d}{dx}\left[\ln\left(\frac{x^2+1}{x+1}\right)\right]_{x=0}$
C. Let $C = \frac{d}{dx}\left[x\lfloor\sqrt{x}\rfloor\right]_{x=2021}$
D. If $f(x) = \sqrt{x} - \sqrt[3]{x}$ for $x \geq \frac{1}{2}$, let $D = \frac{d}{dx}[f^{-1}(x)]_{x=0}$

Compute $A + B + C + D$.

February Regional — Calculus Team — Question #3.

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Compute $A + B + C + D$.

February Regional Calculus Team Question #4.

A. Let A be the rate of change of the circumference of a circle (in inches per second) with an area of 4π in^2 that is increasing at a rate of π in^2/sec.
B. Let B be the rate of change of the area of a square (in square inches per second) with a perimeter of 8 in that is increasing at a rate of 4 in/sec.
C. Let C be the rate of change of the area of a regular hexagon (in square inches per second) with a side length of 1 in that is increasing at a rate of 2 in/sec.
D. Let D be the rate of change of the surface area of a cube (in square inches per second) with a volume of 27 in^3 that is increasing at a rate of 3 in^3/sec.

Compute $ABCD$.

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Compute $ABCD$.

There exist real numbers $m < n < p < q$ such that the function $f(x) = Me^{mx} + Ne^{nx} + Pe^{px} + Qe^{qx}$ satisfies the equation $f''''(x) - 3f''(x) + f(x) = 0$ for all M, N, P, Q, and x. For each of the following parts, express your answer in the form $a + b\sqrt{c}$ for rational a, b and square-free positive integer c.

A. Let $A = |m|$.
B. Let $B = |n|$.
C. Let $C = |p|$.
D. Let $D = |q|$.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #5.**

There exist real numbers $m < n < p < q$ such that the function $f(x) = Me^{mx} + Ne^{nx} + Pe^{px} + Qe^{qx}$ satisfies the equation $f''''(x) - 3f''(x) + f(x) = 0$ for all M, N, P, Q, and x. For each of the following parts, express your answer in the form $a + b\sqrt{c}$ for rational a, b and square-free positive integer c.

A. Let $A = |m|$.
B. Let $B = |n|$.
C. Let $C = |p|$.
D. Let $D = |q|$.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #6.**

A function f satisfies $f''''(x) = 24$ for all x. Suppose $f(0) = 20$, $f'(0) = 4, f''(0) = -2$, and $f'''(0) = 12$.

A. Let $A = f'''(1)$.
B. Let $B = f''(1)$.
C. Let $C = f'(1)$.
D. Let $D = f(1)$.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #6.**

A function f satisfies $f''''(x) = 24$ for all x. Suppose $f(0) = 20$, $f'(0) = 4, f''(0) = -2$, and $f'''(0) = 12$.

A. Let $A = f'''(1)$.
B. Let $B = f''(1)$.
C. Let $C = f'(1)$.
D. Let $D = f(1)$.

Compute $A + B + C + D$.

Let $f(x) = x^3 + x$. Louis is interested in approximating the value of $I = \int_0^6 f(x)\,\mathrm{d}x$.

A. Let A be the approximation of I resulting from a left-hand Riemann sum with three equal subintervals.
B. Let B be the approximation of I resulting from a right-hand Riemann sum with three equal subintervals.
C. Let C be the approximation of I resulting from a midpoint Riemann sum with three equal subintervals.
D. Let D be the approximation of I resulting from a trapezoidal sum with three equal subintervals.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #7.**

Let $f(x) = x^3 + x$. Louis is interested in approximating the value of $I = \int_0^6 f(x)\,\mathrm{d}x$.

A. Let A be the approximation of I resulting from a left-hand Riemann sum with three equal subintervals.
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C. Let C be the approximation of I resulting from a midpoint Riemann sum with three equal subintervals.
D. Let D be the approximation of I resulting from a trapezoidal sum with three equal subintervals.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #8.**

Let $f(x) = x^4 - 8x^3 + 18x^2 - 16x + 22$.

A. Let A denote the sum of the (complex) roots of f.
B. Let B denote the sum of the x-coordinates of all local extrema of f.
C. Let C denote the y-coordinate of the global minimum of f.
D. Let D denote the sum of the x-coordinates of all points of inflection of f.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #8.**

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C. Let C denote the y-coordinate of the global minimum of f.
D. Let D denote the sum of the x-coordinates of all points of inflection of f.

Compute $A + B + C + D$.

Let P be the point on the graph of $y = \sqrt{x}$ that is closest to the line $y = \frac{1}{4}x + 2$.

A. Let A be the x-coordinate of P.
B. Let B be the y-coordinate of P.
C. Let C be the distance from P to $y = \frac{1}{4}x + 2$.
D. Let D be the slope of the tangent line to $y = \sqrt{x}$ at P.

Compute $ABCD$.

February Regional Calculus Team Question #9.

Let P be the point on the graph of $y = \sqrt{x}$ that is closest to the line $y = \frac{1}{4}x + 2$.

A. Let A be the x-coordinate of P.
B. Let B be the y-coordinate of P.
C. Let C be the distance from P to $y = \frac{1}{4}x + 2$.
D. Let D be the slope of the tangent line to $y = \sqrt{x}$ at P.

Compute $ABCD$.

For a function $f(x)$ and input a, let $L(f(x), a)$ denote the maximum value of δ such that for all x in the domain of f,

$$|x - a| < \delta \Rightarrow |f(x) - f(a)| < 1.$$

You may recognize this as the epsilon-delta definition of continuity for $\varepsilon = 1$.

A. Let $A = L\left(\frac{x}{2}, 2021\right)$.
B. Let $B = L(x^2, 2)$.
C. Let $C = L(\sqrt{x}, 4)$.
D. Let $D = L(2^x, 0)$.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #10.**

For a function $f(x)$ and input a, let $L(f(x), a)$ denote the maximum value of δ such that for all x in the domain of f,

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You may recognize this as the epsilon-delta definition of continuity for $\varepsilon = 1$.

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B. Let $B = L(x^2, 2)$.
C. Let $C = L(\sqrt{x}, 4)$.
D. Let $D = L(2^x, 0)$.

Compute $A + B + C + D$.

## February Regional		Calculus Team		Question #11.

Let K be the cone of maximum lateral surface area that can be inscribed in a sphere of radius 3.

A. Let A be the radius of K.
B. Let B be the height of K.
C. Let C be the slant height of K.
D. Let D be the lateral surface area of K.

Compute $ABCD$.

## February Regional		Calculus Team		Question #11.

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A. Let A be the radius of K.
B. Let B be the height of K.
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Compute $ABCD$.

February Regional **Calculus Team** **Question #12.**

Suppose f is an odd function (meaning $f(-x) = -f(x)$ for all real x), and g is an even function (meaning $g(-x) = g(x)$ for all real x). Suppose further that $\int_0^3 f(x)\,\mathrm{d}x = 5$, $\int_0^5 f(x)\,\mathrm{d}x = 8$, $\int_0^3 g(x)\,\mathrm{d}x = 2$, and $\int_0^5 g(x)\,\mathrm{d}x = 9$.

A. Let $A = \int_0^3 \big(2f(x) + g(x)\big)\,\mathrm{d}x$.
B. Let $B = \int_{-5}^5 f(x)\,\mathrm{d}x$.
C. Let $C = \int_3^5 f(x)\,\mathrm{d}x$.
D. Let $D = \int_{-3}^5 g(x)\,\mathrm{d}x$.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #12.**

Suppose f is an odd function (meaning $f(-x) = -f(x)$ for all real x), and g is an even function (meaning $g(-x) = g(x)$ for all real x). Suppose further that $\int_0^3 f(x)\,\mathrm{d}x = 5$, $\int_0^5 f(x)\,\mathrm{d}x = 8$, $\int_0^3 g(x)\,\mathrm{d}x = 2$, and $\int_0^5 g(x)\,\mathrm{d}x = 9$.

A. Let $A = \int_0^3 \big(2f(x) + g(x)\big)\,\mathrm{d}x$.
B. Let $B = \int_{-5}^5 f(x)\,\mathrm{d}x$.
C. Let $C = \int_3^5 f(x)\,\mathrm{d}x$.
D. Let $D = \int_{-3}^5 g(x)\,\mathrm{d}x$.

Compute $A + B + C + D$.

February Regional Calculus Team Question #13.

Let $f^{(n)}(x)$ denote the nth derivative of f evaluated at x. The *degree-n Maclaurin approximation* of an n-times differentiable function f is given by

$$T_n(x) = \sum_{i=0}^{n} \frac{1}{i!} f^{(i)}(0) x^i.$$

A. If $f(x) = x^4$, let $A = T_4(1)$.
B. If $f(x) = e^x$, let $B = T_4(1)$.
C. If $f(x) = \sin(x)$, let $C = T_4(1)$.
D. If $f(x) = \cos(x)$, let $D = T_4(1)$.

Compute $A + B + C + D$.

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D. If $f(x) = \cos(x)$, let $D = T_4(1)$.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #14.**

Let $f(x) = x^4 e^x + x^3 e^x$.

A. Let $A = f(1)$.
B. Let $B = f'(1)$.
C. Let $C = f''(1)$.
D. Let $D = f'''(1)$.

Compute $A + B + C + D$.

February Regional **Calculus Team** **Question #14.**

Let $f(x) = x^4 e^x + x^3 e^x$.

A. Let $A = f(1)$.
B. Let $B = f'(1)$.
C. Let $C = f''(1)$.
D. Let $D = f'''(1)$.

Compute $A + B + C + D$.