Answer Key:

1. D
2. A
3. A
4. B
5. A

6. D
7. B
8. C
9. A
10. D

11. E
12. A
13. C
14. A
15. D

16. D
17. B
18. D
19. D
20. B

21. B
22. A
23. C
24. C
25. B

26. B
27. D
28. A
29. C
30. D

Solutions:

1. D: Look for a system where the boundary lines are parallel lines – they have the same slope but different y-intercepts. The half-plane of $3y \leq 2x - 8$ is below the boundary line and the half-plane of $y \geq \frac{2}{3}x - 1$ is above the boundary line so the intersection is empty and the system has no solution.

2. A: $4i(1 + 9i)(6 - 2i) = (4i + 36i^2)(6 - 2i) = (-36 + 4i)(6 - 2i) =$

$-216 + 72i + 24i - 8i^2 = -208 + 96i$. The imaginary part is the coefficient of i so it's 96.

3. A: $\sqrt[5]{32x^{11}y^{20}z^5} = \sqrt[5]{2^5 \cdot x^5 \cdot x^5 \cdot x \cdot y^5 \cdot y^5 \cdot y^5 \cdot y^5 \cdot z^5} = 2x^2y^4z \cdot \sqrt[5]{x}$

4. B: Assume that the leading actor is listed first. After that the remaining five actors can be chosen in 5! different ways. The assumption is that the leading actor is listed last. Again the other five actors can be chosen is 5! ways. In total there are 2(5!) orders; and so, 2(120) = 240.

5. A: Find the y-coordinate of the vertex. The x-coordinate is $-\frac{b}{2a} = -\frac{(-5)}{4}$ or $\frac{5}{4}$. The minimum function value is $f\left(\frac{5}{4}\right) = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 8 = \frac{39}{8}$.

6. D: The radicand needs to be non-negative so $30x - x^3 \geq 0$. Factoring leads to

$x(\sqrt{30} - x)(\sqrt{30} + x) \geq 0$. The expression is equal to zero at $x = 0, -\sqrt{30}$, and $\sqrt{30}$.

Values less than $-\sqrt{30}$ and between 0 and $\sqrt{30}$ make the expression positive. So the domain is $(-\infty, -\sqrt{30}] \cup [0, \sqrt{30}]$.

7. B: $\frac{f(a+h)-f(a)}{h} = \frac{[7(a+h)^2-3(a+h)]-[7a^2-3a]}{h} = \frac{[7a^2+14ah+7h^2-3a-3h]-[7a^2-3a]}{h}$

$$= \frac{14ah + 7h^2 - 3h}{h} = 14a + 7h - 3$$

8. C: Factor the numerator and the denominator: $\frac{x^3+4x^2-11x-30}{x^3+7x^2+4x-12} = \frac{(x+2)(x+5)(x-3)}{(x+2)(x-1)(x+6)}$

The graph has two vertical asymptotes at $x = 1, x = -6$. (There is a hole at $x = -2$.) There is a horizontal asymptote at $y = 1$. So there are 2 + 1 = 3 asymptotes.

9. A: The inverse is found by replacing x with y and vice versa. Also in the inverse $y < 5$.

Inverse: $x = (y - 5)^2 + 2$, $y < 5$. Solving for y: $y = 5 \pm \sqrt{x - 2}$.

Since $y < 5$, the $\pm$ will be a subtraction sign. $h^{-1}(x) = 5 - \sqrt{x - 2}$.

10. D: If $f(-x) = f(x)$ for all x in the domain of $f(x)$, then it is an even function.

$f(-x) = 3(-x)^4 + 5(-x) + 9 = 3x^4 + 5x + 9$, so f is even.

$g(-x) = e^{-(-x)^2} = e^{-x^2}, so\ g$ is even.

$h(-x) = -3(-x)^2 - 11 = -3x^2 - 11$. So h is even.

$k(-x) = |(-x)^3| = |-x^3| = |x^3|$. So k is even. All the functions are even.

11. E: Complete the square on both variables:

$9(x^2 - \frac{2}{3}x + \frac{1}{9}\ ) - 15(y^2 - 2y + \ 1\ \ ) = 10 + 1 - 15 \rightarrow 9\left(x - \frac{1}{3}\right)^2 - 15(y - 1)^2 = -4.$
So the center is $\left(\frac{1}{3}, 1\right)$.

12. A: Isolate the square root $2x + 2 = \sqrt{100 - 12x}$ and then square both sides.
$(2x + 2)^2 = \left(\sqrt{100 - 12x}\right)^2 \ \rightarrow\ 4x^2 + 8x + 4 = 100 - 12x \ \rightarrow\ 4x^2 + 20x - 96 = 0.$
$x^2 + 5x - 24 = 0 \ \rightarrow\ (x + 8)(x - 3) = 0$. Only $x = 3$ will solve the original equation. ($x = -8$ is an extraneous solution.)

13. C: Use a substitution $u = e^x$ to show the equation is in quadratic form.
$11u - 39 - 20u^{-1} = 0$. Multiply both sides by u. $11u^2 - 39u - 20 = 0$. Solve by factoring.
$(11u + 5)(u - 4) = 0$. $u = -\frac{5}{11}$ or 4. Since $u = e^x$, u must be positive. $e^x = 4$ so $x = \ln 4$.

14. A: In order for the relation, $c(x)$, to result in a conic section that is a hyperbola, we need an eccentricity that is greater than 1. Thus, we need $e \in \{2, 3, 4 \ldots 9\}$, which give us a probability of $\frac{8}{10} = \frac{4}{5}$.

15. D: In order for the relation, $c(x)$, to result in a conic section that encloses a finite area, we need it to be either a circle or an ellipse. Now, since $0 < e < 1$ is the eccentricity for an ellipse (thus, a non-integer), we only need to consider the possibility of a circle which has eccentricity $e = 0$. This gives us the probability of $\frac{1}{10}$.

16. D: Elements in corresponding positions must be equal. $3x + 4y = x + z$ and $2y - z = 3y - z$ and also, $7 = x + y + z$. $2y - z = 3y - z$ leads to $y = 0$.

Substituting that into the other two equations leads to a 2 by 2 system: $\begin{cases} 3x = x + z \\ x + z = 7 \end{cases} \rightarrow 3x = 7\ \ or\ x = \frac{7}{3}$ and $z = \frac{14}{3}$. Thus, $xy + yz + xz = \frac{7}{3} \times 0 + 0 \times \frac{14}{3} + \frac{7}{3} \times \frac{14}{3} = \frac{98}{9}$.

17. B: The first parabola has a vertex of $(-2, 4)$. The directed distance, p, from its vertex to its directrix is found by solving $4p = 12$. So $p = 3$. Since x was the square variable the parabola opens upward. The directrix is $y = 1$. The second parabola has a vertex of (4, 1). $4p = -8$ or $p = -2$. The parabola opens leftward so the directrix is $x = 6$. The intersection of the directrices is the point (6, 1).

18. D: The distance between (3, 5) and (-2, 6) is $\sqrt{(-5)^2 + (1)^2} = \sqrt{26}$. The distance between (3, 5) and (0, 0) is $\sqrt{(3)^2 + (5)^2} = \sqrt{34}$. The distance between (-2, 6) and (0, 0) is $\sqrt{(-2)^2 + (6)^2} = \sqrt{40}$. The product of the three side lengths is then $\sqrt{26} \times \sqrt{34} \times \sqrt{40} = 4\sqrt{2210}$.

19. D: $\sum_{k=1}^{100}(4k - 7) = (-3) + (1) + (5) + \cdots + (393) = \frac{100(-3+393)}{2} = 19{,}500$.

$\prod_{k=2}^{4} k^2 = (4)(9)(16) = 576$. $19{,}500 + 576 = 20{,}076$.

20. B: Let $x = 1 + \frac{1}{1+\frac{1}{1+\cdots}}$, so the original expression is equivalent to $1 + \frac{1}{x}$.

So $x = 1 + \frac{1}{x}$. Multiplying both sides by x leads to the quadratic equation: $x^2 - x - 1 = 0$.

It has two solutions, $\frac{1\pm\sqrt{5}}{2}$ but original expression is a positive number: $\frac{1+\sqrt{5}}{2}$.

21. B: The average rate of change is the value of $\frac{f(5)-f(2)}{5-2} = \frac{120-9}{3} = 37$.

22. A: $27 = 3^3$, so $N(t) = 4(3^3)^t = 4(3)^{3t} = 4(3)^{t/3}$. The coefficient of t is the tripling time: $\frac{1}{3}$ of an hour or 20 minutes.

23. C: $z = kxy$. Substituting the values $z = 64$, $x = 4$ and $y = 8$, allows us to solve for k.

$64 = k(4)(8) \rightarrow k = 2$. So $z = 2xy$. Now substitute the values $z = 45$ and $y = 4.5$, to solve for x. $45 = 2(x)(4.5)$ or $x = 5$.

24. C: Multiply both sides by the LCD of $x(x + 1)$ which results in the equation: $x(4x + 3) + 2(x + 1) = 1 \rightarrow 4x^2 + 5x + 1 = 0 \rightarrow (4x + 1)(x + 1) = 0$. The values for x are $-\frac{1}{4}$ and -1. However $x = -1$ is an extraneous solution which doesn't solve the original equation.

25. B: Factor the numerator: $-x^2(x^2 - 2x - 8) = -x^2(x - 4)(x + 2)$. Consider all the intervals between $x = -2, x = 0$ and $x = 4$. Solutions are in the intervals: $(-\infty, -2)$, $(-2, 0)$ and $(0, 4)$.

26. B: $\log_k 25 = 2$, so $k^2 = 25$ or $k = 5$. $f\left(\frac{\sqrt{5}}{5}\right) = \log_5\left(\frac{\sqrt{5}}{5}\right) = \log_5(5^{1/2})(5^{-1}) = \log_5(5^{-1/2}) = -\frac{1}{2}$

27. D: Let x be the amount of pure acid needed. The amount of acid in 20L of 30% acid is 6 L. The total amount of acid will be x + 6. Another expression for the total amount of acid in the mixture is 50% of (x + 20) L or 0.5(x + 20). Solving x + 6 = 0.5(x + 20) leads to x = 8.

28. A: Solve $x^2 + 5x - 3 = \pm 7$. Both equations have solutions with a sum of -5, since the linear coefficient of each equation is 5. $-5 + (-5) = -10$.

29. C: Let $(a, -a)$ represent any point on the line $x + y = 0$. The distance between $(a, -a) and$ $(5\sqrt{3}, 5\sqrt{3})$ is $\sqrt{(a - 5\sqrt{3})^2 + (-a - 5\sqrt{3})^2}$. Solve $\sqrt{a^2 - 10\sqrt{3} + 75 + a^2 + 10\sqrt{3} + 75} = 10\sqrt{2}$.

$2a^2 + 150 = 200 \quad \rightarrow \quad a = \pm 5$. The value of a must be negative in order for the point to be in the second quadrant. $(-5, 5)$.

30. D: $y = 3f(2x - 8)$ is a transformation of $y = f(x)$, $(-5, 12)$ vertically stretched by 3 and horizontally by ½ will be the point $\left(-\frac{5}{2}, 36\right)$ and then the graph will be translated to the right 4 units and the point will be at $\left(\frac{3}{2}, 36\right)$.