

1) E

2) C

3) B

4) D

5) A

6) D

7) D

8) B

9) A

10) B

11) D

12) A

13) A

14) C

15) D

16) C

17) D

18) B

19) D

20) E

21) D

22) D

23) D

24) E

25) A

26) C

27) B

28) A

29) D

30) B

- 1) Option A has a square root in it, so if  $k = 2$ , we don't get an integer. The same with Option B. Option D has a  $k$  in the denominator which is bad if  $k = 0$ . Option C is only true if  $k \geq 0$ . So, the answer is **E**.
- 2) Counterexamples for A, B, and D are  $k = 1$ ,  $k = 1$ , and  $k = 0$ , respectively.  $3^{(\text{real number})} > 0$ , so **C** is correct.
- 3)  $0.4^2 = (2/5)^2 = 4/25$ , then the numerator just becomes 1.  $64/16 = 4$  and the square root of  $4 = 2$  and we see the 2s cancel. We end up with  $1/\pi$  and that is pretty close to  $1/3$ . **B**
- 4) Translate this into symbols:  $2x + y = 14$  and  $x - 11y = -108$ . Using the second equation to get an expression for  $x$  we can plug this into the first to get  $2(11y - 108) + y = 14$  which simplifies to  $23y = 230$  or  $y = 10$ . Plug back in to get  $x = 2$ . Thus,  $10/2 = 5$ . **D**
- 5) Just plug in the values that are provided in the answer choices one at a time to find out and when  $x = 2$  is plugged in you can immediately see that this does not equal 0. **A**
- 6) Plug in  $x = 2$  to the original equation and get 7. **D**
- 7) Go letter by letter and subtract the second exponent from the first. If the result is negative, the letter belongs in the denominator, if it's positive it goes in the numerator. For  $a$ ,  $2 - 3 = 1$  in the denominator, for  $b$ ,  $2 - 1 = 1$  in the numerator, and for  $c$ ,  $7 - 3 = 4$  in the numerator. **D**
- 8) Closed means when you perform an operation on two numbers in a set, the result is also in the set. Options 1, 2, and 4 are true. **B**
- 9) The inside expression must either equal 6 or -6. Setting up both equations, we first have  $x^2 + x - 6 = 6$  which becomes  $x^2 + x - 12 = 0 \rightarrow (x - 3)(x + 4) = 0 \rightarrow x = 3, x = -4$ . Next, we have  $x^2 + x - 6 = -6$  which becomes  $x^2 + x = 0 \rightarrow x(x + 1) = 0 \rightarrow x = -1, x = 0$ . The sum of these solutions is -2. **A**
- 10) The ratio of mayo to total volume (both in gallons) is  $M/V = 1/4$  or  $4M = V$ . Then, when we add 1 gallon of water to the mixture, the new ratio is  $M/(V + 1) = 3/20$ . We can use  $4M = V$  to substitute in for the original volume of the mixture and get  $(V/4)/(V + 1) = 3/20$ , cross multiplying we get  $5V = 3V + 3$  which gives us  $V = 1.5$  gallons. **B**
- 11) This is just a fancy way of asking for the lowest common multiple of 2, 7, and 11. Since they are all prime, their LCM is the product of all three:  $2(7)(11) = 154$ . **D**
- 12) The function factors to  $f(x) = (2x - 1)(x - 3)$ , which has zeros  $x = \frac{1}{2}, 3$ . **A**
- 13) Just let  $S(x) = D(x)$  then solve.  $0.25x - 2 = -0.5x + 7 \rightarrow 0.75x = 9 \rightarrow x = 12$ . **A**
- 14) Multiplying the y intercept of  $D(x)$  by 1.75 gives us  $49/4$ , then solving in a similar manner to the previous question, the new price is 19. **C**

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15) This polynomial factors to  $x(x^2 - 9x - 26) = 0$ , which has a factor of  $x$ . This means that one of the roots of the equation is 0, so the product of the three roots must be 0, regardless of what the other two roots are. **D**

16) Add 2 to both sides then factor to  $(x - 2)(x - 5) = 0$  and the largest value is 5. **C**

17)  $g(1) = -1 \rightarrow f(-1) = -4$ .  $f(1) = 2 \rightarrow g(2) = -4$ .  $-4 - (-4) = 0$ . **D**

18) We know that  $A + D = 3187$ ,  $B + C = 2118$ , and  $A + C = 1729$ . Subtract the second two equations to get  $B - A = 389$ . Then, write this in terms of  $A$  and substitute into the first equation to get  $(B - 389) + D = 3187$  or  $B + D = 3576$ . **B**

19) In general, if you know the  $x$  intercept is  $a$  and the  $y$  intercept is  $b$  the formula of that line is  $(x/a) + (y/b) = 1$ . Plugging in we have  $x/(-3) + y/7 = 1$  and multiplying both sides by  $-21$  gives us  $7x - 3y = -21$ . **D**

20) An example of A is the points  $(-1, -1)$  and  $(3, 3)$  which has midpoint  $(1, 1)$ . An example of B is  $(-1, -2)$  and  $(2, 1)$  which has midpoint  $(0.5, -0.5)$ . An example of C is  $(-1, -2)$  and  $(2, 2)$  which has midpoint  $(0.5, 0)$ . And the example of D is when you have any point  $(-a, -a)$  and  $(a, a)$ . All are possible. **E**

21) Multiply  $5(-70) = -350$ , then list out the factors of  $-350$  and look for some that sum to  $-43$  (the middle coefficient).  $7$  and  $-50$  work so we re-write  $5x^2 + 7x - 50x - 70$  and factor by grouping to get  $x(5x + 7) - 10(5x + 7) = (x - 10)(5x + 7)$  for solutions  $-1.4$  and  $10$ . The absolute value of the difference here is  $11.4$ , so we add one for a final answer of  $12.4$ . **D**

22) Each of the first three numbers is pretty close to 1. For the last number, since  $\sqrt{10} > 3$  we have that  $\sqrt{10} - 1 > 3 - 1 = 2$  which is much larger than the others. **D**

23)  $3, 5$ , and  $7$  are all twin primes, as are  $11$  and  $13$ , and  $17$  and  $19$ .  $23$  is not a twin prime, so A is true. There are 8 primes less than  $23$  and  $8^2 = 64$ , so B is true. The 2-digit primes less than  $50$  are  $11, 13, 17, 19, 23, 29, 31, 37, 41, 43$ , and  $47$ . Only  $23$  has a second digit that is one larger than its first digit, so C is true. D is false because  $11, 29, 41, 43$  and  $47$  are all counterexamples. **D**

24) All of these properties are true. A:  $6$  is the first perfect number. B:  $4! = 24$ . C:  $8$  is all three of the listed properties. D:  $3 - 1 = 2, 4 - 1 = 3, 6 - 1 = 5, 8 - 1 = 7, 12 - 1 = 11, 24 - 1 = 23$ , all prime. **E**

25) Square both sides and collect terms/simplify to get  $2x - 2\sqrt{(x - 92)(x + 92)} = 16$ . Then divide by 2 and isolate the root to get  $x - 8 = \sqrt{(x - 92)(x + 92)}$  and square both sides. Using the difference of squares formula, we have  $x^2 - 16x + 64 = x^2 - 92^2$ . We eliminate the  $x^2$  term and just solve the equation  $16x = 92^2 + 64$ . Notice that  $92 = 4 \times 23$  so  $92^2 = 4^2 \times 23^2$ . We can then write the equation as  $16x = 16 \times 23^2 + 16 \times 4$  and cancel the  $16$ 's for a much more manageable number to compute.  $x = 23^2 + 4 = 529 + 4 = 533$ . If you plug back in to the original equation you see that  $533 + 92 = 625$  and

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$533 - 92 = 441$  and those square roots are 25 and 21 so the solution is valid. Otherwise, simple trial and error check on each of the given choices reveals the same answer! **A**

26) We want to find out how many multiples of three there are under 501, so we divide 501 by 3 and subtract one (we're looking at strictly less than, not including 501!) to get 166. Then we do the same for 4's and round down this time get 125. But we have a problem. This counts the multiples of 12 twice (each multiple of twelve appears on the 3 list AND on the 4 list) so we need to subtract away TWICE the number of 12's less than 501. We divide 501 by 12 to get 41 multiples of 12 under 501. Our required sum is then  $166 + 125 - 2 \cdot 41 = 209$ . **C**

27) The quantity  $mn/\text{gcf}(m, n)$  is just the  $\text{lcm}(m, n)$  so we're looking for the quantity  $1 + \text{lcm}(m, n)$ . We do the inside of the parenthesis first:  $1 + \text{lcm}(7, 9) = 64$ . Then, we do  $1 + \text{lcm}(5, 64) = 1 + 320 = 321$ . **B**

28) 
$$\frac{x^{12}+31}{1+31^x} + \frac{(-x)^{12}+31}{1+31^{-x}} = \frac{x^{12}+31}{1+31^x} + \frac{x^{12}+31}{1+\frac{1}{31^x}} = \frac{x^{12}+31}{1+31^x} + \frac{x^{12}+31}{\frac{1+31^x}{31^x}} = \frac{x^{12}+31}{1+31^x} + \frac{31^x(x^{12}+31)}{1+31^x} = \frac{(x^{12}+31)(1+31^x)}{1+31^x} = x^{12} + 31$$
. **A**

29)  $27^2 = 729 < 781 < 28^2 = 784$ , so  $27 < \sqrt{781} < 28$  and 27 is the largest integer that works. **D**

30)  $\sqrt{16} \times \sqrt{9} \times \sqrt{121} = 4 \times 3 \times 11 = 132$ . **B**