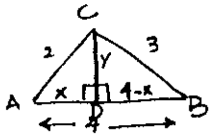
Gaither - Leto Invitational
Geometry Team
Feb. 22, 1947

1. 

$x^2 + y^2 = 4$

$(4-x)^2 + y^2 = 9$

$y^2 = 4 - x^2$

$y^2 = 9 - (4-x)^2 = 9 - (16 - 8x + x^2) = -7 + 8x \quad x^2$

$4 - x^2 = -7 + 8x - x^2$

$11 = 8x$

$\frac{11}{8} = x$

$4 - x = \frac{21}{8}$

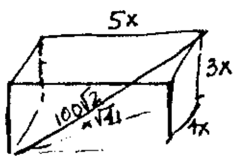
$BD = \frac{21}{8}$

2. slope of $4y + 28 = 8kx \rightarrow -\frac{k}{8}$

slope of $8y + kx = 4 \quad = \frac{2k}{1}$

$\frac{-k}{8_4} \cdot \frac{2k}{1} = -1 \quad \frac{-k^2}{4} = -1 \quad -k^2 = -4 \quad k^2 = 4 \quad k = \pm 2$

3. exterior $\angle = 30°$; $\frac{360}{30} = 12$ sides $\therefore \frac{n(n-3)}{2} = \frac{12(9)}{2} =$ 54 diagonals

4. 

$16x^2 + 25x^2 = 41x^2 \quad$ diagonal $= x\sqrt{41}$

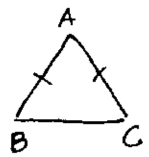
$41x^2 + 9x^2 = 10000 \cdot 2$

$50x^2 = 20{,}000$

$x^2 = 400$

$x = 20 \quad 3x = 60 \cdot 4x = 80, \ 5x = 100$

$3x + 4x + 5x = 60 + 80 + 100 = 240$

5. 

$x + 6 = 2x - 54 \quad m\angle B = 66 \quad BC = 33$

$-x = -60 \quad m\angle C = 66 \quad AB = 22$

$x = 60 \quad AC = 22$

$33 + 22 + 22 = 77$

6. 

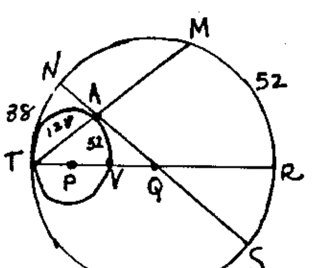
$\frac{x}{3} + \frac{x}{3} + \frac{3x}{3}$

$\frac{5x}{3} = 180$

$5x = 540$

$x = 108$

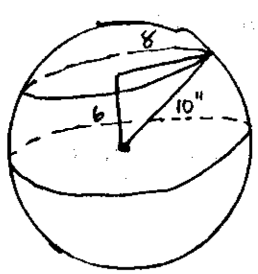
ext. $\angle = 72°$ $\quad \frac{360}{n} = 72 \quad n = 5$

7. 

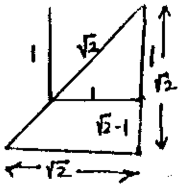
$m\angle ATQ = 26 \quad m\widehat{AV} = 52 \therefore m\widehat{AT} = 128 \quad m\angle AQT = \frac{1}{2}(m\widehat{TA} - m\widehat{AV}) = \frac{1}{2}(128 - 52)$

$= 38 \therefore m\widehat{NT} = 38$ since $\angle AQT$ is a central angle. $m\widehat{TN} + m\widehat{NM} + m\widehat{MR} = 18$

$38 + m\widehat{NM} + 52 = 180 \therefore m\widehat{NM} = 90$

8. 

radius of sphere $= 10'' \therefore 4\pi r^2 = 4\pi\, 100 = 400\pi$



$EB = 2 \therefore 2x^2 = 4$

$x^2 = 2$

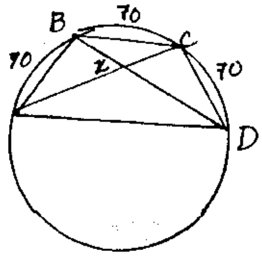
$x = \sqrt{2} = BF = EF$

Area $\square DEF = \frac{1}{2}(\sqrt{2}-1)(1+\sqrt{2})$

$\frac{1}{2}(\sqrt{2}-1+2-\sqrt{2})$

$\frac{1}{2}(1) = \boxed{\frac{1}{2}}$

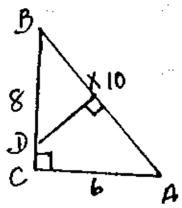
10.



$\overline{BC} \parallel \overline{AD}$, since ABCD is a trapezoid $\therefore m\widehat{AB} = m\widehat{CD} = 70$

$m\angle x = \frac{1}{2}(70+70) = \boxed{70°}$

11.



$\triangle BAC \sim \triangle BDX$ (AA) since $\angle BDX = \angle DXAC$ then $\angle BDX = \frac{1}{2}\angle BAC \therefore$

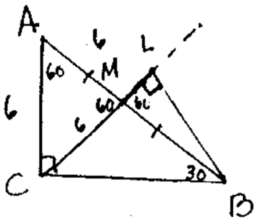
$\frac{BX}{BC} = \frac{1}{\sqrt{2}}$

$\frac{BX}{8} = \frac{1}{\sqrt{2}}$

$\sqrt{2}BX = 8$

$BX = \frac{8}{\sqrt{2}} = \boxed{4\sqrt{2}}$

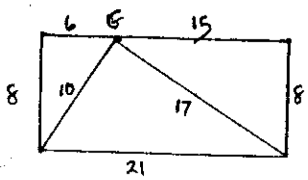
12.



$CM = \frac{1}{2}AB \therefore CM = 6$, since $m\angle B = 30$, $AC = 6 \therefore \triangle ACM$ is equilateral

$m\angle AMC = 60 \quad \therefore m\angle BLC = 60$, $LM = 3$ and $BL = \boxed{3\sqrt{3}}$

13.

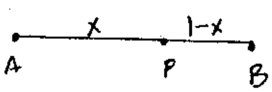


Area $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(14)(7)(3)}$

$\sqrt{72 \cdot 98} = \sqrt{36 \cdot 2 \cdot 49 \cdot 2} = \sqrt{36 \cdot 49 \cdot 4} = 6 \cdot 7 \cdot 2 = \boxed{84}$

14.

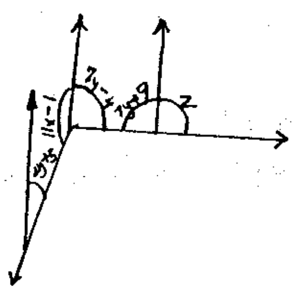


$\frac{1}{x} = \frac{x}{1-x} \quad 1-x = x^2 \quad x^2+x-1=0 \quad x = \frac{-1 \pm \sqrt{1-4(1)(-1)}}{2}$

$= \frac{-1 \pm \sqrt{5}}{2}$

$= \frac{-1+\sqrt{5}}{2}$

15.



$7y - 4 + 7x + 9 = 180$

$11x - 1 + 2y + 5 = 180$

$7x + 7y + 5 = 180$

$11x + 2y + 4 = 180$

$-14x - 14y - 10 = -360$

$77x + 14y + 28 = 1260$

$63x + 18 = 990$

$6x = 882$

$x = 14$

$7x + 9 = 7(14) + 9 = 107 \therefore z = 73$

$7y - 4 = 73$

$7y = 77$

$y = 11$

$x + y + z = 73 + 14 + 11 \boxed{= 98}$