Statistics Team Round Answers – January Statewide 2022

Question	A	B	C	D
1	27.716	27.014	89	85.5
2	1.904	0.034	0.028	1
3	22.732	149.591	8.671	270.267
4	0.427	0.147	16.159	1.350
5	0.707	0	0.097	0.046
6	4	60	2	Iguanas
7	18	15	24	16
8	$\frac{6}{4165}$	$\frac{33}{16660}$	$\frac{1}{259896}$	$\frac{1}{649740}$
9	$\frac{9}{4}$ or 2.25	$\frac{1}{4}$ or 0.25	0	$\frac{31}{160}$ or 0.19375
10	0.290	0.276	0.337	0.092
11	S	P	H	W
12	0.017	0.061	0.029	0.729
13	36	180	40	$\frac{13}{28}$
14	$\frac{1}{4}$	71	82	0

1. A) 27.716 B) 27.014 C) 89 D) 85.5

A) We can enter our data into a list and do 1-Var stats on the calculator. We notice that we want the sample standard deviation and that is the Sx in the calculator, which is 27.716.

B) We have the same scenario, but this time we need the population standard deviation, which is σ on the calculator, which is 27.014.

C) We can sort the list in our calculator and see that 89 is the number with the highest frequency.

D) We can find Q_1 and Q_3 from 1-Var stats, and we can use the $1.5(IQR)$ rule to find any outliers. We see that $1.5(IQR) = 74.25$, so there are clearly no outliers, because $Q_1 - 74.25$ is negative, and $Q_3 + 74.25$ is well over 100. So, we just add Q_1, Q_3, and 0.

2. A) 1.904 B) 0.034 C) 0.028 D) 1

A) We can plug in the given statistics into the *1-Prop Z Test* program to see that the test statistic is z = 1.904 when rounded to 3 decimal places.

B) We know that for a 1-Proportion Z Test, $z = \frac{\hat{p}-P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}}$, where $\hat{p}$ is the sample proportion and P_0 is the hypothesized population proportion. We weed the denominator of the above expression: $\sqrt{\frac{P_0(1-P_0)}{n}} = \sqrt{\frac{0.37(1-0.37)}{200}} = 0.034$ when rounded.

C) We see from our test we did in part A that the p value is 0.028 when rounded.

D) We can see that the p value of 0.028 is less than our alpha level of 0.03, so we reject the null hypothesis, so D is 1.

3. A) 22.732 B) 149.591 C) 8.671 D) 270.267

A) We know that the formula for margin of error of a T-Interval is $ME = t^* \frac{s}{\sqrt{n}}$. We can find t^* by using $t^* = invT(0.935, 14) = 1.608666741$ (remember that if we want an 87% confidence interval, we need 93.5% of the area to the left of the desired critical value because we need 6.5% on both sides; further, the degrees of freedom for this t-interval are $df = n - 1 = 14$). We also know $s = 27.36490627$ from running the *1-Var Stats* program on a list containing the given data, and n is 15. Knowing this, we can find the margin of error using the above formula which is $ME = 1.608666741 \frac{27.36490627}{\sqrt{15}} = 11.36617709$, and doubling it will give the width of the interval, which comes out to be 22.732 when rounded.

B) We have a very similar situation, but this time, our $t^* = invT(0.97, 14) = 2.046169055$. After finding the margin of error using the same method as described in part A (which comes out to be about 14.458), we can add this to the mean of the set, which we know is 135.1333333 from the *1-Var Stats* output, to get our final answer of 149.591 when rounded.

C) We can do the same thing as in part A, but t^* is now $invT(0.88, 14) = 1227157331$. We can use the formula again to see that our final answer is 8.671 when rounded.

D) We know that the sum of the upper and lower bound of any confidence interval is simply 2 times the point estimate used to construct the interval, in this case the sample mean; so, our answer is simply $2 * 135.13333$ which is 270.267 when rounded.

4. A) 0.427 B) 0.147 C) 16.159 D) 1.350

A) The probability that Eddie's session lasts for less than 10 games is $geomcdf(.06, 9) = 0.427$ when rounded because Eddie's scenario is modeled by a geometric random variable.
B) The probability Eric gets more than 2 scores of 100 is the same as 1 – (the probability that he gets 2, 1, or 0 scores of 100) in his 15 plays. This is $1 - binomcdf(15, .09, 2) = 0.147$ when rounded. Alternatively, it is $1 - .91^{15} - 15 * .91^{14} * .09 - 105 * .91^{13} * .09^2 = 0.147$.
C) The standard deviation of a geometric distribution is $\frac{\sqrt{1-p}}{p}$. Plugging in 0.06 and rounding, we get 16.159.
D) The mean number of times eric scores 100 is the mean of a binomial distribution, which is $15 * .09$, which is 1.350.

5. A) 0.707 B) 0 C) 0.097 D) 0.046

A) This probability is equal to $normalcdf(125, 134, 128, 4)$ which is 0.707.
B) The probability of any individual randomly selected value from a continuous distribution is always 0.
C) The height of the density curve at X = 127 is $normalpdf(127, 128, 4)$ = 0.097.
D) Further than two standard deviations away from the mean in either direction means the value must be either less than 120 or greater than 136. To find this probability, we can use the complement rule: $1 - normalcdf(120, 136, 128, 4)$ = 0.046.

6. A) 4 B) 60 C) 2 D) Iguanas

A) The number of treatments is equal to the number of factors times number of levels for each factor. There are two factors and two levels for each factor: the old or new shock rod, and whether or not the rod is dipped in water. So, there is a total of 4 treatments.
B) She needs 15 iguanas for each treatment group. There are 4 treatment groups, so she needs a total of 60 iguanas.
C) As noted in part A, there are two factors: the old or new shock rod, and whether or not the rod is dipped in water.
D) The experimental units are the things that are being experimented upon. In this case, Helena is experimenting on iguanas, so those are the experimental units.

7. A) 18 B) 15 C) 24 D) 16

For parts A, B, and C, we can set up a 3-variable system of equations with Venn diagram to solve for the parts we need. Let's call the number of ghglorgs who like ooblies and ooblees only x, the number of ghglorgs who like ooblies and ooblides only y, and the number of ghglorgs who like ooblees and ooblides only z. Then, $x + y + 60 + 3 = 102, x + z + 50 + 3 = 86$, and $y + z + 54 + 3 = 99$. Solving this system, we get $x = 15, y = 24$, and $z = 18$. So, part A is 18, part B is 15, and part C is 24. For part D, we simply add up all the parts in our Venn diagram, which is 224. Kira surveyed 240 ghglorgs, so that means 16 ghglorgs like none of the things.

8. A) $\frac{6}{4165}$ B) $\frac{33}{16660}$ C) $\frac{1}{259896}$ D) $\frac{1}{649740}$

A) We know the denominator is $\binom{52}{5}$ because we have a 5-card hand. To find the numerator, we first need to choose two different ranks, which is $\binom{13}{1}\binom{12}{1}$. Then, from there, we need three form one rank and two from the other, which is $\binom{4}{3}\binom{4}{2}$. Evaluating, we get $\frac{6}{4165}$.

B) The denominator is also $\binom{52}{5}$. This time, we need to choose one of the suits, which is $\binom{4}{1}$, and then we need 5 cards from that suit, which is $\binom{13}{5}$. Evaluating, we get $\frac{33}{16660}$.
C) We need a straight flush of clubs. The denominator is once again $\binom{52}{5}$. The numerator is 10 because the straight flush can start from and ace all the way up to a 10. Evaluating, we see our answer is $\frac{1}{259896}$.
D) We know that there are only 4 royal straight flushes (one for each suit). Our denominator is still $\binom{52}{5}$, so our final answer is $\frac{1}{649740}$.

9. A) $\frac{9}{4}$ or 2.25 B) $\frac{1}{4}$ or 0.25 C) 0 D) $\frac{31}{160}$ or 0.19375
A) We can set up an equation to find a. The area of this distribution when x is less than 0.5 is a triangle with area $\frac{(0.5)(0.5)}{2}$, or $\frac{1}{8}$. The area of the rectangle is $0.5(a-0.5)$, which must equal $\frac{7}{8}$ in order for the distribution to be valid. Solving for *a*, we get *a* is $\frac{9}{4} = 2.25$.
B) This probability is the area of the triangle when x is less than 0.5 and the area of the rectangle when x is between .5 and .75. This area is equal to $\frac{1}{8} + .25(.5)$, which is equal to $\frac{1}{4}$ or 0.25.
C) The probability of any one selected value is 0.
D) This probability is the area of the triangle when x is less than 0.5 plus the area of the rectangle when x is between 0.5 and 0.7 minus the area of the triangle when x is less than 0.25. This is equal to $\frac{1}{8} + 0.2(0.5) - \frac{(0.25)(.025)}{2}$, which comes out to $\frac{31}{160}$ or 0.19375.

10. A) 0.290 B) 0.276 C) 0.337 D) 0.092
A) We see that Eddie can win in 4, 5, 6, or 7 games. If we consider four games, the probability of Eddie winning is 0.4^4. If we consider five games, the probability is $4 * 0.4^4 * 0.6$ (the 4 in the front is the ways you can arrange Eddie's first four games because he can choose one of those to lose). For six games, the probability is $10 * 0.4^4 * 0.6^2$. For seven games, the probability is $20 * 0.4^4 * 0.6^3$. Summing all of these and rounding, we get 0.290.
B) We can have either Eric or Eddie win, but the series must last 7 games. That means for the first six games, Eric and Eddie must win 3 apiece, and the last game doesn't matter. The probability of that happening is $20 * 0.4^3 * 0.6^3$, or 0.276.
C) Eric can win in 4 or 5 games. For 4 games, the probability is 0.6^4. For 5 games, the probability is $4 * 0.6^4 * 0.4$. Adding these, we get 0.337.
D) We already found this in part A and is simply $10 * 0.4^4 * 0.6^2$, or 0.092

11. A) S B) P C) H D) W
A) The relationship William is describing is for 2 quantitative variables. The only 2 quantitative variable graph listed is the scatterplot.
B) Bailey wants to display percentages, which would be done best in a pie chart.
C) Jack wants to examine the shape of the heights of people in History class. The best graph listed to examine the shape of data is a histogram.
D) Since Alice's choice of spread refers to IQR and range, the graph best suited to display the shape of her data is a box and whisker plot.

12. A) 0.017 B) 0.061 C) 0.029 D) 0.729

A) This probability is equal to $1 - binompdf(10, 0.17, 4)$. This is 1 minus the probability that 4 or less of his friends are feeling festive. This comes out to be 0.017.

B) This probability is equal to 0.83^{15}, which is 0.061.

C) This probability is 0.17^2, or 0.029.

D) An easier way to calculate this probability is to find the probability that none of his 7 friends are feeling festive, and then take the compliment of that probability. This is equal to $1 - 0.83^7$, which is 0.729.

13. A) 36 B) 180 C) 40 D) $\frac{13}{28}$

A) The number of distinct permutations of the letters in this word is $\frac{13!}{3!2!2!2!}$, which comes out to 129,729,600. The sum of digits of this number is 36.

B) We can treat all the vowels as a "super letter." So, our new word has 5 letters: T H T R EAE. The number of ways we can arrange these are $\frac{5!}{2!}$, which we then multiply by $\frac{3!}{2!}$ for the number of ways to arrange the vowels in our "super letter." This final number comes out to 180.

C) There are 2 ways to put the vowels at the ends of the word: either the 'O' is at the front or the 'E' is at the front and the other letter is at the end. Then, we have to arrange the other 5 letters, which is $\frac{5!}{3!} = 20$. Multiplying by 2 gives us our answer of 40.

D) If a 'Z' is at the front, there are $\frac{7!}{2!}$ ways to arrange the other letters. There are the same number of arrangements with a 'Z' at the end (think about reversing the order of the letters in the word). However, we have double-counted cases where there is a 'Z' at both the front and the end. The number of cases for that is $\frac{6!}{2!}$. So, the number of cases where there is a 'Z' at either the front or the end is $7! - \frac{6!}{2} = 4680$. The total number of ways to arrange the letters is $\frac{8!}{2!2!} = 10080$. Dividing these numbers gives us a probability of $\frac{13}{28}$.

14. A) $\frac{1}{4}$ B) 71 C) 82 D) 0

A) If Mr. Lu wants the standard deviation to be a quarter of what it is now, he has to divide all the scores by 4. So, A is $\frac{1}{4}$.

B) After Mr. Lu divides all the scores by 4, the new mean score is 14. He wants the final mean score to be 85, so he has to add 71 to each score after dividing them each by 4. So, B is 71.

C) To find Olivia's new score, we have to divide her old score by 4 and then add 71, which gives us 82.

D) To find Zach's old score, we have to subtract 71 from his new score and then multiply by 4, which gives us 0.