

#1 Precalculus team
January Statewide 2022

Given the parabola $4y = x^2 + 2x + 9$:

- A. What is the sum of the coordinates of the vertex?
- B. What is the sum of the coordinates of the focus?
- C. The equation of the directrix of the parabola is in the form $y = k$. What is k ?
- D. What is the length of the latus rectum?

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#2 Precalculus team
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- A. What is the coefficient of the xy^2 term in the expansion of $(x + 2y)^3$?
- B. What is the coefficient of the x^3y^3 term in the expansion of $(3x - y)^6$?
- C. What is the sum of the coefficients in the expansion of $(2x + y)^5$?
- D. What is the constant term in the expansion of $(x^2 + \frac{2}{x})^6$?

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#3 Precalculus team
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Given that $\sin(\alpha) = \frac{3}{5}$ and $\cos(\beta) = \frac{12}{13}$, and $0 < \alpha, \beta < \frac{\pi}{2}$

- A. What is $\sin(\alpha + \beta)$?
- B. What is $\cos(\alpha - \beta)$?
- C. What is $\sin(2\alpha)$?
- D. What is $\tan(\beta - \alpha)$?

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#4 Precalculus team
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How many distinct real roots do each of the following equations have?

A. $x^2 - 12x + 35 = 0$

B. $x^3 + x^2 - 2 = 0$

C. $x^4 + 3x^2 + 2 = 0$

D. $x^4 - 8x^3 + 24x^2 - 32x + 16 = 0$

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#5 Precalculus team
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- A. Simplify $\sqrt{432} + \sqrt{108} + \sqrt{588}$.
- B. How many integer values are in the domain of $y = \sqrt{40 - 2x^2 + 2x}$?
- C. What is the maximum value of the function $y = 44 + 8\sqrt{11 + 14\sin(58854x + \pi e^5)}$?
- D. Simplify $\sqrt{9 + 4\sqrt{5}}$.

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#6 Precalculus team
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- A. What is the sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$?
- B. What is the sum of the first 10 perfect squares? (1 is the first perfect square)
- C. What is the 12th triangular number?
- D. What is the sum of the series $\sum_{\theta=0}^{360} \cos(\theta^\circ)$?

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#7 Precalculus team
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- A. What is the value of the determinant of the matrix $\begin{bmatrix} 8 & 11 \\ 7 & 13 \end{bmatrix}$?
- B. What is the value of the determinant of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ -2 & 5 & -3 \\ 0 & 7 & -1 \end{bmatrix}$?
- C. What is the sum of the elements in the inverse of the matrix $\begin{bmatrix} 3 & 6 \\ -2 & 3 \end{bmatrix}$?
- D. What is the determinant of the minor of the 2nd row, 3rd column element of $\begin{bmatrix} 4 & -1 & 2 \\ 6 & 0 & -5 \\ 3 & 1 & -2 \end{bmatrix}$?

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#8 Precalculus team
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- A. What is the probability of getting exactly 1 head when flipping a fair coin 3 times?
- B. What is the probability of getting at least 1 head when flipping a fair coin 3 times?
- C. Senioritis is running rampant at Buchholz! There is a 60% chance a randomly selected person has senioritis at Buchholz. 20% of the people at Buchholz are seniors. Given that being a senior and having senioritis are independent, what is the probability that a randomly selected person is a senior and has senioritis?
- D. I have an unfair coin with a probability of $p > \frac{1}{2}$ of flipping heads. If the probability I flip exactly one head out of two flips with this coin is .375, then what is the probability of flipping tails with this coin?

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#9 Precalculus team
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- A. Simplify $(\log_4 343)(\log_7 16)$.
- B. Simplify $2^{\log_4 25}$.
- C. $3^{\ln x} = 7^{\ln 3}$. What is x ?
- D. How many times does the graph of $\log_{10} x$ intersect the graph of $x^2 - 2$?

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#10 Precalculus team
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- A. What is $\arcsin\left(\sin\left(\frac{3\pi}{2}\right)\right)$?
- B. What is $\tan(\arccos x)$?
- C. Simplify $32 \sin\left(\frac{\pi}{128}\right) \left(\cos\left(\frac{\pi}{128}\right) \cos\left(\frac{\pi}{64}\right) \cos\left(\frac{\pi}{32}\right) \dots \cos\left(\frac{\pi}{4}\right)\right)$.
- D. Simplify $\left(\frac{\cos^3(x) - \sin^3(x)}{\cos(2x)}\right) (1 + \tan(x)) \left(\frac{\cos(x)}{1 + \frac{1}{2}\sin(2x)}\right)$.

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#11 Precalculus team
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- A. What is the area of a circle with a circumference of 20π ?
- B. What is the area of an equilateral triangle with a side length of 6?
- C. What is the area of a rectangle with integer side lengths and a diagonal of length $\sqrt{61}$?
- D. What is the area of a regular octagon with a side length of 3?

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#12 Precalculus team
January Statewide 2022

- A. What is the length of the major axis of the ellipse $x^2 + 2y^2 - 4x + 4y = 10$?
- B. Find the sum of the x and y coordinates of both foci of the ellipse $9x^2 + 4y^2 - 54x + 32y + 109 = 0$.
- C. What is the largest solution to the equation $x^4 - 12x^3 + 33x^2 - 2x - 48$?
- D. Given that $x + \frac{1}{x} = 3$, what is $x^3 + \frac{1}{x^3}$?

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#13 Precalculus team
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- A. How many distinct arrangements are there of 8 books on a bookshelf if 3 of the books are red, 3 of them are blue, and the other 2 are green? (Two books are taken to be indistinguishable if they are the same color).
- B. I pick 2 cards out of a standard 52 card deck without replacement. What is the probability the first one is a king and the second one is a spade?
- C. Zach has an urn with 20 white balls and 5 red balls. How many red balls does he need to add to the urn to have a 75% chance of picking a red ball out of the urn?
- D. A bug starts on a number line at 0. Every second, he either moves one unit left or one unit right with equal probability. What is the probability he returns to 0 on the number line in 4 seconds or less?

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#14 Precalculus team
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- A. Triangle ABC has side $AB = 5$, side $BC = 7$, and side $AC = 8$. An angle bisector is drawn from B to side AC, and it intersects side AC at point D. What is the length of segment AD?
- B. What is the diameter of the circumscribed circle of a triangle with side lengths of 14, 48, and 50?
- C. What is the inradius of a triangle with side lengths 13, 14, and 15?
- D. Point P is placed in rectangle ABCD such that segment $AP = 5$, segment $BP = 4$, and segment $CP = 7$. What is the length of segment DP?

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