Question	A	B	C	D
1	1	2	1	4
2	12	-540	243	240
3	$56/65$	$63/65$	$24/25$	$-16/63$
4	2	1	0	1
5	$32\sqrt{3}$	10	84	$2+\sqrt{5}$
6	$1/2$	385	78	1
7	27	-10	$2/21$	7
8	$3/8$	$7/8$	$3/25$	$1/4$
9	6	5	7	2
10	$-\frac{\pi}{2}$	$\frac{\sqrt{1-x^2}}{x}$	$\frac{1}{2}$	1
11	100π	$9\sqrt{3}$	30	$18+18\sqrt{2}$
12	8	-2	8	18
13	560	$1/52$	55	$5/8$
14	$10/3$	50	4	$\sqrt{58}$

1.
Completing the square on the right side, we get $4y = (x+1)^2 + 8$. Moving the 8 over, we get $4(y-2) = (x+1)^2$. This parabola is in the form $4p(y-k) = (x-h)^2$ where p is the distance between the vertex and focus as well as the distance between the vertex and directrix, 4p is the length of the latus rectum, and the vertex is (h,k).

A h+k=-1+2=1
B Since it is upwards facing, the coordinate of the focus is (h,k+p), so it's h+k+p=2
C Since it is upwards facing, the directrix is p units below the vertex, so it's y=1, so the answer is 1.
D The length of the latus rectum is 4p, which is 4.

2.
A $3C2(x)(2y)^2 = 12$.
B $6C3(3x)^3(-y)^3 = -540$.
C The sum of the coefficients is the same where $x = 1$ and $y = 1$. So, plugging this in, $(3)^5 = 243$.
D $6C2(x^2)^2(2/x)^4 = 240$.

3.
With the given information, we also get $cos(\alpha) = \frac{4}{5}$ and $sin(\beta) = \frac{5}{13}$ by simple Pythagorean triplets. This also gives us $tan(\alpha) = \frac{3}{4}$ and $tan(\beta) = \frac{5}{12}$
A $sin(\alpha+\beta) = sin(\alpha)cos(\beta) + cos(\alpha)sin(\beta)$. Plugging in the values, we get $\frac{56}{65}$.
B $\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$. Plugging the values, we get $\frac{63}{65}$.
C $sin(2\alpha) = 2sin(\alpha)cos(\alpha)$. Plugging in, we get $\frac{24}{25}$.
D $tan(\beta-\alpha) = \frac{tan(\beta)-tan(\alpha)}{1+tan(\beta)tan(\alpha)}$. Plugging in the values, we get $\frac{-16}{63}$.

4.
A The quadratic factors to $(x-7)(x-5)$, so there are 2 distinct real roots.
B By observation, 1 is a root. After synthetically dividing the function, you get $x^2 + x + 2$. Since the discriminant is 0, there are no other roots, meaning there is only 1 real root.
C Say $a = x^2$. The given equation turns into $a^2 + 3a + 2$ which is $(a+1)(a+2)$. Since a cannot be a negative number, this has 0 solutions.
D This is $(x-2)^4$, so it has 1 distinct solution.

5.
A $432 = 144 * 3, 108 = 36 * 3, 588 = 196 * 3$. So, this turns into $12\sqrt{3} + 6\sqrt{3} + 14\sqrt{3} = 32\sqrt{3}$
B Take out a root 2, and the function under the square root becomes $20 + \mathrm{x} - \mathrm{x}^2$ which factors into $(5+x)(4-x)$. Since it is under a square root, the expression must be non-negative. Taking the critical points of $x = -5$ and $x = -4$, we see this is where $-5 \le x \le 4$. So, there are 10 integers.

C The maximum of sin is 1, so we have $44 + 8 * \sqrt{25} = 84$

D Since it is 2sqrt 20, we hope the test writer doesn't suck and that this can be expressed as $\sqrt{\left(\sqrt{a}+\sqrt{b}\right)^2}$. Where $a + b = 9$ and $ab = 20$. Saying $b = \frac{20}{a}$, and plugging that into the first equation, we have $a + \frac{20}{a} = 9$. Multiplying both sides by a, we have $a^2 + 20 = 9a$.Moving that over, we have $a^2 - 9a - 20$. Factoring, we have $(a - 5)(a - 4)$, so we know that a and b are 4 and 5, doesn't matter which is which. So, $2 + \sqrt{5}$.

6

A This is an infinite geometric sum with first term $\frac{1}{3}$ and common ratio $\frac{1}{3}$. Therefore, the sum is $\frac{a}{1-r}$ where a is the first term and r is the common ratio. So, the sum is $\frac{1}{2}$.

B You can either add the first 10 perfect squares or use the formula $\frac{n(n+1)(2n+1)}{6}$ where $n = 10$. So, that's $\frac{10(11)(21)}{6} = 385$.

C The nth triangle number is just the sum of n integers. So, the 12^{th} triangular number is $\frac{n(n+1)}{2}$ where $n = 12$ which is 78.

D The sum of cosines around the unit circle is 0. However, since there is a 0 and 360, there is a 1 leftover

7

A $8 * 13 - 7 * 11 = 27$.

B We can use expansion by minors. This gives us $3\big((-1 * 5) - (-3 * 7)\big) - 1(2) + 4(-14) = -10$

C The inverse of a 2x2 matrix is to swap the elements in the main diagonal and negate the other ones, then divide each element by the determinant of the original matrix. The determinant is 21, and the sum of 3,3, -6, and 2 is 2, so $\frac{2}{21}$

D The minor of an element is the determinant of the resulting matrix when the row and column of the element are deleted. If we delete the row and column of the element in the 2^{nd} row and 3^{rd} column, we end up with $\begin{bmatrix} 4 & -1 \\ 3 & 1 \end{bmatrix}$, which has a determinant of 7.

8

A There are 3 ways to decide which flip you get the heads. The chance of getting 1 head and 2 tails is $3(.5)(.5)^2$, which comes out to $\frac{3}{8}$.

B To make it easier, we can do 1 – P(no heads). The probability of no heads is $\left(\frac{1}{2}\right)^3$, which is $\frac{1}{8}$. Therefore the probability of getting at least one head is $1 - \frac{1}{8} = \frac{7}{8}$.

C Since the events are independent, the probability of both events occurring is just P(senior)*P(has senioritis). So, the probability is .12.

D From the given information, we have the equation $2p(1-p) = \frac{3}{8}$. Multiplying both sides by 8, we get $16p - 16p^2 = 3$. Moving everything to the right side and factoring, we have $(4p-1)(4p-3)$. Since it says that probability of flipping heads is $> \frac{1}{2}, p = \frac{3}{4}$. So, the probability of flipping tails in one flip is $\frac{1}{4}$.

9

A We can use change of base, to turn the question into $\frac{log343}{log4} * \frac{log16}{log7}$. This is the same as $\frac{3log7}{2log2} * \frac{4log2}{log7} = 6$.

B Since $2 = 4^{\frac{1}{2}}$, we can rewrite the expression as $4^{log_4 5}$ which is 5.

C $a^{lnb} = b^{lna}$ so x=7. For the proof of the first expression, just log both sides.

D By graphing it, it is clear that there are 2 solutions.

10

A $sin(3\pi/2) = -1$. Arcsin is defined from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, so the arcsin of -1 is $-\frac{\pi}{2}$.

B $cos(\theta) = x$, so the sides of the triangle are x, $\sqrt{1-x^2}$, and 1. Tangent of the angle is $\frac{\sqrt{1-x^2}}{x}$.

C If we take just the first two terms, $sin\left(\frac{\pi}{128}\right)$ and $cos\left(\frac{\pi}{128}\right)$, we can put them together to make $\frac{1}{2}sin\left(\frac{\pi}{64}\right)$. We can keep doing this until we get $sin\left(\frac{\pi}{4}\right)$ and $cos\left(\frac{\pi}{4}\right)$ When we combine these, we have combined 6 sine and cosine functions, meaning our coefficient is $\frac{1}{64} * 32 = \frac{1}{2}$. $\frac{1}{2} * sin\left(\frac{\pi}{2}\right) = \frac{1}{2}$.

D $\frac{cos^3(x) - sin^3(x)}{cos^2(x) - sin^2(x)} = \frac{(cos(x) - sin(x))\left(cos^2(x) - cos(x)sin(x) + sin^2(x)\right)}{(cos(x) - sin(x))(cos(x) + sin(x))}$.. This simplifies to $\frac{1 + \frac{1}{2}sin(2x)}{\cos(x) + \sin(x)}$. In the third fraction, there is a $1 + \frac{1}{2}sin(2x)$ on the bottom, so now we have $\frac{1}{cosx + sinx} * (1 + tan(x)) * cos(x)$. Simplifying $1 + \tan(x)$, this is $\frac{\sin(x) + \cos(x)}{\cos(x)}$. So, everything cancels, and this is 1.

11

A If the circumference is 20π, the radius is 10. Therefore, the area is 100π.

B The area of an equilateral triangle with side length s is $\frac{s^2\sqrt{3}}{4}$. Plugging 6 in for s, we get $9\sqrt{3}$.

C The only sums of squares of integers that sum to 61 is $5^2 + 6^2$. So, the sides of the rectangle are 5 and 6, and the area is 30.

D An octagon can be dissected into 4 isosceles right triangles, 4 rectangles and a square by drawing lines between the diagonals. The area of one right triangle is $\frac{s^2}{4}$, the area of one rectangle is $\frac{s^2\sqrt{2}}{2}$, and the area of the center square is s^2. Adding all of these areas up gives us a total area of $2s^2(1+\sqrt{2})$. Plugging 3 into this formula gives us $18 + 18\sqrt{2}$.

12

A Completing the square for x and y on the left you get $(x-2)^2+2(y+1)^2=16$. Dividing both sides by 16, the equation is $\frac{(x-2)^2}{16}+\frac{(y+1)^2}{8}=1$. This is in the form of an ellipse, where a is half of the major axis and a^2=16. So, a=4 and the length of the major axis is 8.
B The sum of the foci is twice the sum of the coordinates of the center. This is because it is a coordinate +- c, so when adding them, the cs cancel out. Since the center is (3, -4), the sum of the coordinates of the foci is -2.
C First, we see that -1 is a root. Synthetically dividing this in, we get $x^3-13x^2+46x-48$. From this, we see that 2 is a root and synthetically divide it. We get $x^2-11x-24$ from this. From here, we factor and get $(x-8)(x-3)$, so the biggest root is 8.
D So $x^3+\frac{1}{x^3}$ factors to $\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$.To find $x^2+\frac{1}{x^2}$, we square $x+\frac{1}{x}$, giving us $x^2+\frac{1}{x^2}+2=9$, so $x^2+\frac{1}{x^2}=7$. Plugging this into what we're looking for, we get $3*6=18$.

13

A We just need the permutations of the 8 books. This is $\frac{8!}{3!3!2!}$, which is 560.
B There are 2 cases. Either the king is a spade, or it is not. There is a $\frac{1}{52}$ chance the king is the king of spades, and then there is a $\frac{12}{51}$ chance of getting a spade on the next draw. There is a $\frac{3}{52}$ chance the king is not spades, and then a $\frac{13}{51}$ chance of getting a spade on the next draw. Multiplying these probabilities and adding, we get $\frac{1}{52}$.
C The probability right now that he gets a red ball is $\frac{1}{5}$. Every red ball we add adds one more to the numerator and denominator. We can write an inequality: $\frac{5+x}{25+x}\geq\frac{3}{4}$. Cross multiplying, we get $20+4x\ \geq 75+3x$. Solving, we get $x\ \geq 55$, so he needs at least 55.
D After the first second, the ant is 1 unit away. Then, there is a $\frac{1}{2}$ chance that the ant goes back to 0 after the 2nd second where it's done, and there's a $\frac{1}{2}$ chance the ant goes 2 away from 0. Now, to get back to 0, he must go back towards 0 twice, which is a $\frac{1}{2}*\frac{1}{2}=\frac{1}{4}$ chance. So, the total probability is $\frac{1}{2}+\frac{1}{2}*\frac{1}{4}=\frac{5}{8}$.

14

A We can call AD x, and then CD is 8-x. Using angle bisector theorem, $\frac{5}{x}=\frac{7}{8-x}$. Cross multiplying, we get $40-5x=7x$. Solving, we get $x=\frac{10}{3}$.
B This is a right triangle, so the hypotenuse is the diameter of the circle. So, the diameter is 50.
C The formula for the inradius is $A=sr$, where r is the inradius, s is the semiperimeter of the triangle, and A is the area. We can solve for the area using heron's formula, which is 84. The semiperimeter is $\frac{13+14+15}{2}$, which is 21. Plugging in, we get the inradius is 4.

D This uses the British Flag Theorem, which says if a point P is placed in a rectangle ABCD, then $AP^2 + CP^2 = BP^2 + DP^2$. Plugging in the 3 lengths we have, we get $DP^2 = 58$, and DP is $\sqrt{58}$.