

1. What is the amplitude of  $\tan(x)$ ?  
A. 1            B. 2            C.  $\pi$             D. 2021            E. NOTA
2. How many positive palindromes are there less than 2021?  
A. 116            B. 117            C. 118            D. 119            E. NOTA
3. What is the sum of the prime factors of 2021?  
A. 57            B. 90            C. 2021            D. 2022            E. NOTA
4. The sum of the amplitude, phase shift, and period of  $y = 2517\sin(3x+6) + 2021$  is in the form  $\frac{a\pi}{b} + c$  where  $a$ ,  $b$ , and  $c$  are all integers and  $a$  and  $b$  are relatively prime with  $b$  being positive. What is  $a - b + c$ ?  
A. 2026            B. 2519            C. 2520            D. 2526            E. NOTA
5. "I was born the youngest baby ever" - Zach Hilman. However, Alex disagrees, and believes he was born the youngest baby ever. Right now, Alex is twice the age of Zach. In 10 years, Alex will be 6 years older than Zach. In 5 years, how old will Alex be?  
A. 8            B. 12            C. 13            D. 17            E. NOTA
6. Kira is starting a garden! The border of her garden is defined by the equation  $36x^2 + 25y^2 - 72x + 100y - 764 = 0$ . What is the area she has available to garden in?  
A.  $30\pi$             B.  $36\pi$             C.  $150\pi$             D.  $900\pi$             E. NOTA

7. Find the number of distinct permutations in the word SLICKRICK.
- A. 40320      B. 45360      C. 90720      D. 362880      E. NOTA
8. How many points  $(x, y)$  exist such that  $4x + 5y = 193$  and  $x$  and  $y$  are positive integers?
- A. 7      B. 8      C. 9      D. 10      E. NOTA
9. Mr. Lu is whipping up some of his famous Mu Lu Juice. Right now, he has 20 liters of a solution that is 35% Mu Lu Juice and 65% water. He adds 30 liters of a different solution of Mu Lu Juice and water. He now has 50 liters of a solution that is 20% Mu Lu Juice and 80% water. What percentage of the 30 liters of solution was Mu Lu Juice?
- A. 5%      B. 7%      C. 10%      D. 20%      E. NOTA
10. What is the sum  $\sum_{n=2}^{\infty} \frac{1}{n^2+n-2}$ ?
- A.  $\frac{5}{9}$       B.  $\frac{11}{18}$       C.  $\frac{5}{3}$       D.  $\frac{11}{6}$       E. NOTA
11. What is the value of  $\cos(13131^\circ) \sin(4869^\circ) + \sin(13131^\circ) \cos(4869^\circ)$ ?
- A. 0      B.  $\frac{\sqrt{2}}{2}$       C.  $\frac{\sqrt{3}}{2}$       D. 1      E. NOTA
12. Triangle ABC has sides  $AB = 30$ ,  $BC = 72$  and  $AC = 78$ . Point D is placed on side BC such that  $BD = 40$  and  $CD = 32$ . What is  $\sin(\angle CAD)$ ?
- A.  $\frac{16}{65}$       B.  $\frac{4}{13}$       C.  $\frac{4}{5}$       D.  $\frac{12}{13}$       E. NOTA

13. How many asymptotes does the function  $\frac{x^3+x^2-10x+8}{x^2+x-6}$  have?

- A. 1      B. 2      C. 3      D. 5      E. NOTA

14. Simplify  $(2 \cos^2(x) - 1) \left( \frac{2 \tan(x)}{\frac{\cos(2x)}{\cos^2(x)}} \right)$ , where defined.

- A.  $\cos(x)$       B.  $\cos(2x)$       C.  $\sin(x)$       D.  $\sin(2x)$       E. NOTA

15. What is the domain of the function  $\sqrt{\log_{\pi}(\log_3(\log_5(x)))}$ ?

- A.  $[5, \infty)$       B.  $[\pi^3, \infty)$       C.  $[125, \infty)$       D.  $[\pi^{125}, \infty)$       E. NOTA

16. What is an equation of an asymptote of the equation  $2x^2 - 3y^2 + 4x - 18y - 31 = 0$ ?

- A.  $y = \frac{2}{3}x + \frac{11}{3}$       B.  $y = \frac{4}{9}x + \frac{31}{9}$   
C.  $y = -\frac{4}{9}x + \frac{23}{9}$       D.  $y = -\frac{2}{3}x + \frac{7}{3}$       E. NOTA

17. How many times do the graphs  $y = \sin(x)$  and  $y = \frac{x}{100}$  intersect?

- A. 61      B. 62      C. 63      D. 64      E. NOTA

18. Right triangle ZLU with a right angle at L has side ZL = 7 and side LU = 24. Point W is placed inside the triangle such that WZ = 5 and  $m\angle WZU = 60^\circ$ . What is the length of UW?

- A.  $15\sqrt{2}$       B.  $5\sqrt{19}$       C.  $10\sqrt{5}$       D.  $5\sqrt{21}$       E. NOTA

19. What is the area of the shape bound by the equation  $|x| + |y| < 32$ ?

- A. 32      B. 64      C. 1024      D. 2048      E. NOTA

20. Simplify  $\frac{\sin^2(x) + \cos(2x) + 2\cos^2(x)}{3\cos(x)}$ , where defined.

- A.  $\cos(x)$       B.  $\sin(x)$       C.  $\cos(2x)$       D.  $3\cos(x)$       E. NOTA

21. Jack has 3 coins with an equal chance of landing on either side. One coin has tails on both sides, one has heads on one side and tails on the other, and one has heads on both sides. He picks one at random and flips it. Given it is heads, what is the probability that the next flip of that coin is also heads?

- A.  $\frac{1}{3}$       B.  $\frac{1}{2}$       C.  $\frac{2}{3}$       D.  $\frac{5}{6}$       E. NOTA

22. How many of these functions are one-to-one?

- I.  $y = \ln(\ln(x))$
- II.  $y = \cos(\sin(x))$
- III.  $y = 2(x - 1)^3$
- IV.  $y = x \cdot 2^x$
- V.  $y = 5$

- A. 2      B. 3      C. 4      D. 5      E. NOTA

23. Given  $f(x) = \frac{9x^2 + 30x + 29}{3x + 5}$ , what is  $f(2021)$ , rounded to the nearest integer?

- A. 2021      B. 6068      C. 6069      D. 18,219      E. NOTA

24. What is the value of the determinant of the matrix  $\begin{bmatrix} 3 & 7 & 5 \\ 2 & 4 & 1 \\ 9 & 6 & 7 \end{bmatrix}$ ?
- A. -124      B. -89      C. -56      D. -19      E. NOTA
25. Circle O has two chords AB and CD that intersect at point E. AE is 12, BE is 2, and CE is 6. AB and CD intersect at a right angle. How far away is point E from the center of circle O?
- A. 1      B.  $\sqrt{24}$       C. 5      D.  $\sqrt{26}$       E. NOTA
26. Jeffery is playing on a game show where he is trying to win a car. There are 4 doors, one of which has a car behind it and the other three of which have trash cans behind them. Jeffery picks a door. Then, the host opens a door to reveal a trash can behind it and offers Jeffery the option to switch doors if he wants to. If he does not switch, what is the probability that the door he chose has the car behind it?
- A.  $\frac{1}{4}$       B.  $\frac{1}{3}$       C.  $\frac{3}{8}$       D.  $\frac{2}{3}$       E. NOTA
27. What is the sum of the digits of the greatest prime divisor of  $11^4 + 324$ ?
- A. 5      B. 10      C. 13      D. 23      E. NOTA
28. Triangle ABC has angles  $A = 120^\circ$ ,  $B = 45^\circ$ , and  $C = 15^\circ$ . The angle bisector of angle A is drawn and extended to side BC. It intersects side BC at point D, and  $AD = 6$ . What is the area of triangle ABD? (hint: you will need  $\sin(75^\circ)$  or  $\sin(15^\circ)$  in your computations. Consider angle addition or subtraction to find these.)
- A.  $\frac{27-9\sqrt{3}}{2}$       B.  $\frac{9\sqrt{6}-9\sqrt{2}}{2}$       C.  $\frac{9\sqrt{6}+9\sqrt{2}}{2}$       D.  $\frac{27+9\sqrt{3}}{2}$       E. NOTA

29. Eric is having trouble finding the inverse of a matrix. Help him by finding the inverse of the matrix  $\begin{bmatrix} 18 & 42 \\ 15 & 35 \end{bmatrix}$ .

A.  $\begin{bmatrix} 35 & -42 \\ -15 & 18 \end{bmatrix}$

B.  $\begin{bmatrix} 7 & -\frac{42}{5} \\ -3 & -\frac{18}{5} \end{bmatrix}$

C.  $\begin{bmatrix} -18 & 15 \\ 42 & -35 \end{bmatrix}$

D.  $\begin{bmatrix} \frac{7}{2} & -\frac{21}{5} \\ -\frac{3}{2} & -\frac{9}{5} \end{bmatrix}$

E. NOTA

30. Mr. Frazer is cutting team sets from a sheet of paper. Every time he makes a cut, he separates one team set from the piece of paper. It takes him 2 minutes to separate a sheet of paper with 4 team sets on it. How long would it take him to separate a sheet of paper with 10 team sets on it?

A. 4

B. 5

C. 6

D. 8

E. NOTA