

#1 Geometry Team
January State-wide 2021

Let PQR be a right triangle with right angle at Q . Find the length of \overline{PR} if:

- A) $PQ = 5$ and $QR = 12$
- B) $PQ = 40$ and $QR = 42$
- C) $QR = 2$ and $PR = PQ + 1$
- D) $PQ = 6$ and the area of PQR is numerically equal to the perimeter of PQR

#1 Geometry Team
January State-wide 2021

Let PQR be a right triangle with right angle at Q . Find the length of \overline{PR} if:

- A) $PQ = 5$ and $QR = 12$
- B) $PQ = 40$ and $QR = 42$
- C) $QR = 2$ and $PR = PQ + 1$
- D) $PQ = 6$ and the area of PQR is numerically equal to the perimeter of PQR

#2 Geometry Team
January State-wide 2021

- A) The complement of angle A and the supplement of angle A are supplementary. Find the degree measure of angle A .
- B) Five times the complement of angle B is equal to two times the supplement of angle B . Find the degree measure of angle B .
- C) An interior angle of a regular polygon with C sides has a measure of 162° . Find C .
- D) Let D be an acute angle measuring x° , where x is a positive integer. If the supplement of the complement of D and the complement of D are supplementary, find the sum of all possible values of x .

#2 Geometry Team
January State-wide 2021

- A) The complement of angle A and the supplement of angle A are supplementary. Find the degree measure of angle A .
- B) Five times the complement of angle B is equal to two times the supplement of angle B . Find the degree measure of angle B .
- C) An interior angle of a regular polygon with C sides has a measure of 162° . Find C .
- D) Let D be an acute angle measuring x° , where x is a positive integer. If the supplement of the complement of D and the complement of the supplement of D are supplementary, find the sum of all possible values of x .

#3 Geometry Team
January State-wide 2021

For all integers $n \geq 3$, let $D(n)$ be the number of diagonals in a regular n -gon and let $E(n)$ be the measure of an exterior angle (in degrees) in a regular n -gon.

- A) Find $D(1003)$.
- B) Find $D(1006) - D(1003)$
- C) Find $D(1003)E(1003)$.
- D) Find the positive integer $k \geq 3$ such that $\frac{E(k)}{D(k)}$ is an integer.

#3 Geometry Team
January State-wide 2021

For all integers $n \geq 3$, let $D(n)$ be the number of diagonals in a regular n -gon and let $E(n)$ be the measure of an exterior angle (in degrees) in a regular n -gon.

- A) Find $D(1003)$.
- B) Find $D(1006) - D(1003)$
- C) Find $D(1003)E(1003)$.
- D) Find the positive integer $k \geq 3$ such that $\frac{E(k)}{D(k)}$ is an integer.

#4 Geometry Team
January State-wide 2021

Let ABC be a right triangle with right angle at B . Suppose that $AB = 3$ and $BC = 4$. Points P , Q , and R lie on segments AB , AC , and BC respectively.

- A) If P is the midpoint of \overline{AB} , Q is the midpoint of \overline{AC} , and R is the midpoint of \overline{BC} , find the area of PQR .
- B) If P is the midpoint of \overline{AB} , Q is the midpoint of \overline{AC} , and R is the midpoint of \overline{BC} , find the perimeter of PQR .
- C) If $BPQR$ is a square, find the length of \overline{AQ} .
- D) If $BPQR$ is a square, find the length of \overline{BP} .

#4 Geometry Team
January State-wide 2021

Let ABC be a right triangle with right angle at B . Suppose that $AB = 3$ and $BC = 4$. Points P , Q , and R lie on segments AB , AC , and BC respectively.

- A) If P is the midpoint of \overline{AB} , Q is the midpoint of \overline{AC} , and R is the midpoint of \overline{BC} , find the area of PQR .
- B) If P is the midpoint of \overline{AB} , Q is the midpoint of \overline{AC} , and R is the midpoint of \overline{BC} , find the perimeter of PQR .
- C) If $BPQR$ is a square, find the length of \overline{AQ} .
- D) If $BPQR$ is a square, find the length of \overline{BP} .

#5 Geometry Team
January State-wide 2021

Consider the points $P = (x, 3)$, $Q = (0, 0)$, and $R = (4, 0)$ such that P lies in the first quadrant.

- A) Find the area of PQR .
- B) Find the smallest value of x such that the slope of PQ and the slope of PR have a product of -3 .
- C) Find the smallest possible value of $PQ^2 + PR^2$.
- D) Find the measure of $\angle PRQ$, in degrees, when $x = 1$.

#5 Geometry Team
January State-wide 2021

Consider the points $P = (x, 3)$, $Q = (0, 0)$, and $R = (4, 0)$ such that P lies in the first quadrant.

- A) Find the area of PQR .
- B) Find the smallest value of x such that the slope of PQ and the slope of PR have a product of -3 .
- C) Find the smallest possible value of $PQ^2 + PR^2$.
- D) Find the measure of $\angle PRQ$, in degrees, when $x = 1$.

#6 Geometry Team
January State-wide 2021

Consider the following four statements:

- I. If Ash beats Gary, then Ash will become the champion.
- II. If Ash does not beat Gary, then Ash will not become the champion.
- III. If Ash becomes the champion, then Ash beat Gary.
- IV. If Ash does not become the champion, then Ash did not beat Gary.

Your answer to each part should be the roman numeral corresponding to the correct statement.

- A) Find the contrapositive of the converse of statement I.
- B) Find the inverse of the contrapositive of statement I.
- C) If statement I is true, which other statement **must** also be true?
- D) Find the inverse of the converse of the contrapositive of statement I.

#6 Geometry Team
January State-wide 2021

Consider the following four statements:

- I. If Ash beats Gary, then Ash will become the champion.
- II. If Ash does not beat Gary, then Ash will not become the champion.
- III. If Ash becomes the champion, then Ash beat Gary.
- IV. If Ash does not become the champion, then Ash did not beat Gary.

Your answer to each part should be the roman numeral corresponding to the correct statement.

- A) Find the contrapositive of the converse of statement I.
- B) Find the inverse of the contrapositive of statement I.
- C) If statement I is true, which other statement **must** also be true?
- D) Find the inverse of the converse of the contrapositive of statement I.

#7 Geometry Team
January State-wide 2021

Let $ABCD$ be a parallelogram.

- A) If $\angle ADC = 100^\circ$, find the measure of $\angle DAB$ in degrees.
- B) If diagonals AC and BD intersect at P , find the value of $\frac{AC}{AP}$.
- C) Find the perimeter of $ABCD$ if $AC = 12$, $BD = 16$, and $AC \perp BD$.
- D) Point E is chosen on segment AD so that $\angle ABE = 30^\circ$ and $\angle DCE = 20^\circ$. Find the measure of $\angle BEC$, in degrees.

#7 Geometry Team
January State-wide 2021

Let $ABCD$ be a parallelogram.

- A) If $\angle ADC = 100^\circ$, find the measure of $\angle DAB$ in degrees.
- B) If diagonals AC and BD intersect at P , find the value of $\frac{AC}{AP}$.
- C) Find the perimeter of $ABCD$ if $AC = 12$, $BD = 16$, and $AC \perp BD$.
- D) Point E is chosen on segment AD so that $\angle ABE = 30^\circ$ and $\angle DCE = 20^\circ$. Find the measure of $\angle BEC$, in degrees.

#8 Geometry Team
January State-wide 2021

Let ABC be a triangle. Find the number of possible **integer** values for the length of side BC if:

- A) $AB = 4$ and $AC = 7$.
- B) $AB = 21$ and $AC = 29$.
- C) $AB = 9$, $AC = 12$, and $\angle BAC$ is obtuse.
- D) $AB = 49$, $AC = 50$, and triangle ABC is acute.

#8 Geometry Team
January State-wide 2021

Let ABC be a triangle. Find the number of possible **integer** values for the length of side BC if:

- A) $AB = 4$ and $AC = 7$.
- B) $AB = 21$ and $AC = 29$.
- C) $AB = 9$, $AC = 12$, and $\angle BAC$ is obtuse.
- D) $AB = 49$, $AC = 50$, and triangle ABC is acute.

#9 Geometry Team
January State-wide 2021

Let ABC be a triangle, and let D and E be points on sides AB and AC , respectively, so that $DE \parallel BC$.

- A) If $AD = 3$, $BD = 6$, and $DE = 4$, find the length of segment BC .
- B) If $DE = 4$ and $BC = 6$, find the value of $\frac{AD}{DB}$.
- C) If $DE = 5$, $BC = 8$, and triangle ABC has perimeter 24, find $AD + AE$.
- D) Let F be the point on segment BC such that $DF \parallel AC$. If $AD = 3$, $BD = 6$, and $BF = 8$, find the length of segment DE .

#9 Geometry Team
January State-wide 2021

Let ABC be a triangle, and let D and E be points on sides AB and AC , respectively, so that $DE \parallel BC$.

- A) If $AD = 3$, $BD = 6$, and $DE = 4$, find the length of segment BC .
- B) If $DE = 4$ and $BC = 6$, find the value of $\frac{AD}{DB}$.
- C) If $DE = 5$, $BC = 8$, and triangle ABC has perimeter 24, find $AD + AE$.
- D) Let F be the point on segment BC such that $DF \parallel AC$. If $AD = 3$, $BD = 6$, and $BF = 8$, find the length of segment DE .

#10 Geometry Team
January State-wide 2021

Consider the points $A = (0, 3)$ and $B = (3, 7)$.

- A) Find the length of segment AB .
- B) Find the area of the triangle with vertices at A , B , and $(0,0)$.
- C) Segment AB is reflected about the x -axis and then reflected about the y -axis to obtain a new segment CD . Find the distance between the midpoint of segment AB and the midpoint of segment CD .
- D) Let P be a point on the x -axis. Find the smallest possible value of $PA + PB$.

#10 Geometry Team
January State-wide 2021

Consider the points $A = (0, 3)$ and $B = (3, 7)$.

- A) Find the length of segment AB .
- B) Find the area of the triangle with vertices at A , B , and $(0,0)$.
- C) Segment AB is reflected about the x -axis and then reflected about the y -axis to obtain a new segment CD . Find the distance between the midpoint of segment AB and the midpoint of segment CD .
- D) Let P be a point on the x -axis. Find the smallest possible value of $PA + PB$.

#11 Geometry Team
January State-wide 2021

Let ABC be a triangle with side lengths $AB = 26$, $AC = 30$, and $BC = 28$. The bisector of $\angle BAC$ intersects BC at D , the bisector of $\angle ABC$ intersects AC at E , and lines AD and BE intersect at I .

- A) Find the length of segment BD .
- B) Find the length of segment AE .
- C) Find $\frac{CD}{CE}$.
- D) Find $\frac{AI}{ID}$.

#11 Geometry Team
January State-wide 2021

Let ABC be a triangle with side lengths $AB = 26$, $AC = 30$, and $BC = 28$. The bisector of $\angle BAC$ intersects BC at D , the bisector of $\angle ABC$ intersects AC at E , and lines AD and BE intersect at I .

- A) Find the length of segment BD .
- B) Find the length of segment AE .
- C) Find $\frac{CD}{CE}$.
- D) Find $\frac{AI}{ID}$.

#12 Geometry Team
January State-wide 2021

Let ABC be an isosceles triangle with $AB = AC$.

- A) Let D be the point on side BC such that AD bisects $\angle BAC$. If $BC = 6$, find $BD \cdot CD$.
- B) Suppose the measure of one angle in triangle ABC is twice the measure of another angle in triangle ABC . Find the sum of all possible values of $\angle BAC$, in degrees.
- C) If $AB = x + 1$, $BC = x$, and the distance from A to BC is $x - 1$, find the value of x .
- D) Let P be a point on the altitude from A to BC such that $AP = CP$. If $\angle ABC = 80^\circ$, find the measure of $\angle BPC$, in degrees.

#12 Geometry Team
January State-wide 2021

Let ABC be an isosceles triangle with $AB = AC$.

- A) Let D be the point on side BC such that AD bisects $\angle BAC$. If $BC = 6$, find $BD \cdot CD$.
- B) Suppose the measure of one angle in triangle ABC is twice the measure of another angle in triangle ABC . Find the sum of all possible values of $\angle BAC$, in degrees.
- C) If $AB = x + 1$, $BC = x$, and the distance from A to BC is $x - 1$, find the value of x .
- D) Let P be a point on the altitude from A to BC such that $AP = CP$. If $\angle ABC = 80^\circ$, find the measure of $\angle BPC$, in degrees.

#13 Geometry Team
January State-wide 2021

Let $ABCD$ be a square and let P be a point on side CD . Points W, X, Y, Z are chosen such that W lies on segment PA , X lies on segment PB , Y lies on segment PC , Z lies on segment PD , and $WXYZ$ is a square.

- A) If $WX = 3$ and $AB = 9$, find $\frac{PW}{PA}$.
- B) If $\angle WXP = 60^\circ$, find the measure of $\angle PBC$, in degrees.
- C) If $DP = 5$ and $AP = 13$, find the length of segment BP .
- D) If $DZ = 2$, $ZP = 1$, and $YC = 4$, find the length of segment WX .

#13 Geometry Team
January State-wide 2021

Let $ABCD$ be a square and let P be a point on side CD . Points W, X, Y, Z are chosen such that W lies on segment PA , X lies on segment PB , Y lies on segment PC , Z lies on segment PD , and $WXYZ$ is a square.

- A) If $WX = 3$ and $AB = 9$, find $\frac{PW}{PA}$.
- B) If $\angle WXP = 60^\circ$, find the measure of $\angle PBC$, in degrees.
- C) If $DP = 5$ and $AP = 13$, find the length of segment BP .
- D) If $DZ = 2$, $ZP = 1$, and $YC = 4$, find the length of segment WX .

#14 Geometry Team
January State-wide 2021

Let ABC be a right triangle with right angle at B such that $AB = 15$ and $BC = 20$. Let D be the point on hypotenuse AC such that $BD \perp AC$ and let E and F be points on AB and AC , respectively, such that $DE \perp AB$ and $DF \perp BC$.

- A) Find the length of segment BD .
- B) Find the length of segment ED .
- C) Find the length of segment EF .
- D) Find AF^2 .

#14 Geometry Team
January State-wide 2021

Let ABC be a right triangle with right angle at B such that $AB = 15$ and $BC = 20$. Let D be the point on hypotenuse AC such that $BD \perp AC$ and let E and F be points on AB and AC , respectively, such that $DE \perp AB$ and $DF \perp BC$.

- A) Find the length of segment BD .
- B) Find the length of segment ED .
- C) Find the length of segment EF .
- D) Find AF^2 .