#1 Calculus Team
January Statewide 2022

Let $f(x) = \frac{x^2+3x-7}{x+1}$ and $g(x) = 2x^3 + 18x^2 + 41x + 20$

A) Find $\frac{d}{dx}\left(f\left(\frac{7}{3}\right)\right)$.
B) Find $f''(1)$.
C) Let $h(x) = g(f(x))$. Find $h'(-4)$.
D) Find the slope of the line normal to $g(x)$at its point of inflection.

#1 Calculus Team
January Statewide 2022

Let $f(x) = \frac{x^2+3x-7}{x+1}$ and $g(x) = 2x^3 + 18x^2 + 41x + 20$

A) Find $\frac{d}{dx}\left(f\left(\frac{7}{3}\right)\right)$.
B) Find $f''(1)$.
C) Let $h(x) = g(f(x))$. Find $h'(-4)$.
D) Find the slope of the line normal to $g(x)$at its point of inflection.

#2 Calculus Team
January Statewide 2022

Let $f(x) = (x-3)^2 + 5$. Approximate the area bounded by $f(x), y = 0, x = 1$, and $x = 5$ using:

A) A left-hand Riemann sum with four intervals of equal size
B) A right-hand Riemann sum with four intervals of equal size
C) A midpoint Riemann sum with four intervals of equal size
D) A trapezoidal sum with four intervals of equal size

#2 Calculus Team
January Statewide 2022

Let $f(x) = (x-3)^2 + 5$. Approximate the area bounded by $f(x), y = 0, x = 1$, and $x = 5$ using:

A) A left-hand Riemann sum with four intervals of equal size
B) A right-hand Riemann sum with four intervals of equal size
C) A midpoint Riemann sum with four intervals of equal size
D) A trapezoidal sum with four intervals of equal size

#3 Calculus Team
January Statewide 2022

All cups are initially full, the people sip their beverages at a constant rate of $0.5\ unit^3/second$, and the people need not tilt their cups in order to drink from them. Assume units are in the appropriate combination of $units$ and $seconds$.

A) Mr. Payne drinks coconut water from a spherical coconut of radius 3. What volume of coconut water remains when the height of the remaining coconut water is decreasing most slowly?
B) Mr. Moody drinks Dr. Pepper from a cylindrical can of radius 1.5 and height 4.5. At what rate in is the height of his remaining drink changing $10\ seconds$ after he starts drinking from it?
C) Mrs. Frazer drinks water from a conical cup of radius 2 and height 4 with its vertex pointed at the ground. With regards to her remaining water, at what rate is its height changing with respect to its volume when its height is 2?
D) Mr. Frazer drinks sweet tea from a frustum-shaped cup of bottom radius 1, rim radius 2, and height 8. At what rate is the height of his remaining tea changing when the height is 2?

#3 Calculus Team
January Statewide 2022

All cups are initially full, the people sip their beverages at a constant rate of $0.5\ unit^3/second$, and the people need not tilt their cups in order to drink from them. Assume units are in the appropriate combination of $units$ and $seconds$.

A) Mr. Payne drinks coconut water from a spherical coconut of radius 3. What volume of coconut water remains when the height of the remaining coconut water is decreasing most slowly?
B) Mr. Moody drinks Dr. Pepper from a cylindrical can of radius $1.5$and height 4.5. At what rate in is the height of his remaining drink changing $10\ seconds$ after he starts drinking from it?
C) Mrs. Frazer drinks water from a conical cup of radius 2 and height 4 with its vertex pointed at the ground. With regards to her remaining water, at what rate is its height changing with respect to its volume when its height is 2?
D) Mr. Frazer drinks sweet tea from a frustum-shaped cup of bottom radius 1, rim radius 2, and height 8. At what rate is the height of his remaining tea changing when the height is 2?

#4 Calculus Team
January Statewide 2022

Consider the following seven functions:

$f(x) = x^2 + 2x + 1$	$j(x) = 2^{-x}$	$m(x) = \cot^{-1}(x)$
$g(x) = x^2 + 4x + 4$	$k(x) = \log_3(x)$	
$h(x) = 3f(x) - g(x)$	$l(x) = \sin(2x)$	

A) For how many of the above functions is the derivative increasing at $x = \frac{2\pi}{3}$?
B) How many of the above functions are increasing and concave down at $x = \frac{\pi}{8}$?
C) How many of the above functions are decreasing and concave up at $x = -\frac{\pi}{3}$?
D) For how many of the above functions is the magnitude of their slope decreasing at $x = \frac{7\pi}{12}$?

#4 Calculus Team
January Statewide 2022

Consider the following seven functions:

$f(x) = x^2 + 2x + 1$	$j(x) = 2^{-x}$	$m(x) = \cot^{-1}(x)$
$g(x) = x^2 + 4x + 4$	$k(x) = \log_3(x)$	
$h(x) = 3f(x) - g(x)$	$l(x) = \sin(2x)$	

A) For how many of the above functions is the derivative increasing at $x = \frac{2\pi}{3}$?
B) How many of the above functions are increasing and concave down at $x = \frac{\pi}{8}$?
C) How many of the above functions are decreasing and concave up at $x = -\frac{\pi}{3}$?
D) For how many of the above functions is the magnitude of their slope decreasing at $x = \frac{7\pi}{12}$?

#5 Calculus Team
January Statewide 2022

Let $f(x) = \frac{1}{1+x^3}$. Find

A) $f(0)$
B) $f'(0)$
C) $f''(0)$
D) $f'''(0)$

#5 Calculus Team
January Statewide 2022

Let $f(x) = \frac{1}{1+x^3}$. Find

A) $f(0)$
B) $f'(0)$
C) $f''(0)$
D) $f'''(0)$

#6 Calculus Team
January Statewide 2022

Let $x > 0$ be a real number. Find the maximum possible value of

A) $x + \frac{4}{x}$
B) $-x^3 + 6x^2 + 15x - 31$
C) $x^2 e^{-x}$
D) $\tan\left(\arctan\left(\frac{12}{x}\right) - \arctan\left(\frac{3}{x}\right)\right)$

#6 Calculus Team
January Statewide 2022

Let $x > 0$ be a real number. Find the maximum possible value of

A) $x + \frac{4}{x}$
B) $-x^3 + 6x^2 + 15x - 31$
C) $x^2 e^{-x}$
D) $\tan\left(\arctan\left(\frac{12}{x}\right) - \arctan\left(\frac{3}{x}\right)\right)$

#7 Calculus Team
January Statewide 2022

Let $f(x)$ be a continuous and differentiable function over the reals. Some values of $f(x)$ are shown in the chart below.

x	-4	-1	0	3	5
$f(x)$	-3	6	8	-1	24

A) Rolle's Theorem can be applied correctly to $f(x)$on the interval $(0,r)$, where r is some constant. What is $f(r)$?
B) $f(x)$ has the property that $f'(x) \geq 0$ on the interval $[-4,0]$. If $M$is the maximum value of $f(x)$ on the interval and m is the minimum value of $f(x)$ on the interval, then what is $M - m$? If either Mor m does not necessarily exist or does not necessarily have a fixed value, replace it with 0 in the expression.
C) If, given that w and v must be values of x shown in the chart, the shortest open interval over which $f(x)$ is guaranteed to have a zero is(w,v), find $w + v$.
D) Let r be a value of x guaranteed by the Mean Value Theorem for Derivatives over the interval (s,t), where s and t are values of x shown in the chart. The slope of the tangent line to $f(x)$ at $x = r$ is -3. Find $s + t$?

#7 Calculus Team
January Statewide 2022

Let $f(x)$ be a continuous and differentiable function over the reals. Some values of $f(x)$ are shown in the chart below.

x	-4	-1	0	3	5
$f(x)$	-3	6	8	-1	24

A) Rolle's Theorem can be applied correctly to $f(x)$on the interval $(0,r)$, where r is some constant. What is $f(r)$?
B) $f(x)$ has the property that $f'(x) \geq 0$ on the interval $[-4,0]$. If $M$is the maximum value of $f(x)$ on the interval and m is the minimum value of $f(x)$ on the interval, then what is $M - m$? If either Mor m does not necessarily exist or does not necessarily have a fixed value, replace it with 0 in the expression.
C) If, given that w and v must be values of x shown in the chart, the shortest open interval over which $f(x)$ is guaranteed to have a zero is(w,v), find $w + v$.
D) Let r be a value of x guaranteed by the Mean Value Theorem for Derivatives over the interval (s,t), where s and t are values of x shown in the chart. The slope of the tangent line to $f(x)$ at $x = r$ is -3. Find $s + t$?

#8 Calculus Team

January Statewide 2022

The following functions are continuous and differentiable everywhere. All letters represent constants except for x and the function names.

$f(x) = \begin{cases} a_1x^2 + b_1x + 12 & x \le 3 \\ a_1x + b_1 & x > 3 \end{cases}$	$h(x) = \begin{cases} 3x^2 + 2x + 13 & x < c_3 \\ d_3x + 1 & x \ge c_3 \end{cases}$
$g(x) = \begin{cases} a_2x^3 + b_2x^2 + c_2x & x < 1 \\ 8 & x \ge 1 \end{cases}$	$j(x) = \begin{cases} 4x^2 + 5x + 6 & x < 1 \\ a_4x^3 + b_4x^2 + c_4x + d_4 & 1 \le x \le 2 \\ x^2 + 2x + 3 & x > 2 \end{cases}$

A) Find $a_1 + b_1$
B) Find $2a_2 + b_2$
C) Given that $c_3 < 0$, find c_3d_3
D) Find $a_4 + b_4 + c_4$

#8 Calculus Team

January Statewide 2022

The following functions are continuous and differentiable everywhere. All letters represent constants except for x and the function names.

$f(x) = \begin{cases} a_1x^2 + b_1x + 12 & x \le 3 \\ a_1x + b_1 & x > 3 \end{cases}$	$h(x) = \begin{cases} 3x^2 + 2x + 13 & x < c_3 \\ d_3x + 1 & x \ge c_3 \end{cases}$
$g(x) = \begin{cases} a_2x^3 + b_2x^2 + c_2x & x < 1 \\ 8 & x \ge 1 \end{cases}$	$j(x) = \begin{cases} 4x^2 + 5x + 6 & x < 1 \\ a_4x^3 + b_4x^2 + c_4x + d_4 & 1 \le x \le 2 \\ x^2 + 2x + 3 & x > 2 \end{cases}$

A) Find $a_1 + b_1$
B) Find $2a_2 + b_2$
C) Given that $c_3 < 0$, find c_3d_3
D) Find $a_4 + b_4 + c_4$

#9 Calculus Team
January Statewide 2022

Dan the Dingo is running along the x-axis. At time $t \geq 0$ (in seconds), his position $x(t)$ at time t is given by $x(t) = 2t^2 - \sqrt{t}$.

A) Find the time t at which Dan's velocity is zero.
B) Find the total amount of seconds for which Dan's speed is decreasing.
C) Find the total distance traveled by Dan from $t = 0$ to $t = 9$.
D) Find the minimum possible value of $x(t)$.

#9 Calculus Team
January Statewide 2022

Dan the Dingo is running along the x-axis. At time $t \geq 0$ (in seconds), his position $x(t)$ at time t is given by $x(t) = 2t^2 - \sqrt{t}$.

A) Find the time t at which Dan's velocity is zero.
B) Find the total amount of seconds for which Dan's speed is decreasing.
C) Find the total distance traveled by Dan from $t = 0$ to $t = 9$.
D) Find the minimum possible value of $x(t)$.

#10 Calculus Team
January Statewide 2022

Let $f(x)$ be the piecewise function:

$$f(x) = \begin{cases} x - \sqrt[3]{x^3 + 6x^2 + 7x + 3} & x < 0 \\ 8 & x = 0 \\ 4x + 8 & 0 < x < 2 \\ 5 & x = 2 \\ \left(1 + \frac{2}{x}\right)^{2x} & x > 2 \end{cases}$$

A) Let n equal the number of values of x on $(-\infty, \infty)$ for which $f(x)$ is differentiable but not continuous. Find n.

B) Find $\lim_{x \to 2} f(x)$

C) Find $\lim_{x \to 0} f(x)$

D) Find $\lim_{x \to -\infty} f(x) + \lim_{x \to \infty} f(x)$

#10 Calculus Team
January Statewide 2022

Let $f(x)$ be the piecewise function:

$$f(x) = \begin{cases} x - \sqrt[3]{x^3 + 6x^2 + 7x + 3} & x < 0 \\ 8 & x = 0 \\ 4x + 8 & 0 < x < 2 \\ 5 & x = 2 \\ \left(1 + \frac{2}{x}\right)^{2x} & x > 2 \end{cases}$$

A) Let n equal the number of values of x on $(-\infty, \infty)$ for which $f(x)$ is differentiable but not continuous. Find n.

B) Find $\lim_{x \to 2} f(x)$

C) Find $\lim_{x \to 0} f(x)$

D) Find $\lim_{x \to -\infty} f(x) + \lim_{x \to \infty} f(x)$

#11 Calculus Team
January Statewide 2022

A) Find the maximum possible area of a triangle inscribed in $x^2 + y^2 = 9$.
B) Find the maximum possible area of a triangle inscribed in $4x^2 + 9y^2 = 36$.
C) Find the maximum possible area of a hexagon inscribed in $x^2 + y^2 = 9$.
D) Find the maximum possible area of a hexagon inscribed in $4x^2 + 9y^2 = 36$.

#11 Calculus Team
January Statewide 2022

A) Find the maximum possible area of a triangle inscribed in $x^2 + y^2 = 9$.
B) Find the maximum possible area of a triangle inscribed in $4x^2 + 9y^2 = 36$.
C) Find the maximum possible area of a hexagon inscribed in $x^2 + y^2 = 9$.
D) Find the maximum possible area of a hexagon inscribed in $4x^2 + 9y^2 = 36$.

#12 Calculus Team
January Statewide 2022

Let $f(x) = \int_0^{x^2} te^t dt$ and $g(x) = x^2 \ln(x)$. Find:

A) $f(1)$
B) $f\left(\sqrt{g(2)}\right)$
C) $\frac{d}{dx}\left(f(x)g(x)\right)$ at $x = 1$
D) $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$ at $x = 1$

#12 Calculus Team
January Statewide 2022

Let $f(x) = \int_0^{x^2} te^t dt$ and $g(x) = x^2 \ln(x)$. Find:

A) $f(1)$
B) $f\left(\sqrt{g(2)}\right)$
C) $\frac{d}{dx}\left(f(x)g(x)\right)$ at $x = 1$
D) $\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$ at $x = 1$

#13 Calculus Team
January Statewide 2022

Consider the parabola $y = x^2 + 3$.

A) Find the product of the slopes of the lines passing through $(0, 0)$and tangent to $y = x^2 + 3$.
B) Find the product of the slopes of the lines passing through $(2020, 0)$and tangent to $y = x^2 + 3$.
C) Let θ be the acute angle formed by the two lines passing through $(1, 0)$ and tangent to $y = x^2 + 3$. Find $\tan(\theta)$.
D) Let A and B be points on $y = x^2 + 3$ such that the tangent lines to $y = x^2 + 3$ at A and B intersect at $(2020, 0)$. Line AB intersects the $y -$axis at $(0, c)$. Find c.

#13 Calculus Team
January Statewide 2022

Consider the parabola $y = x^2 + 3$.

A) Find the product of the slopes of the lines passing through $(0, 0)$and tangent to $y = x^2 + 3$.
B) Find the product of the slopes of the lines passing through $(2020, 0)$and tangent to $y = x^2 + 3$.
C) Let θ be the acute angle formed by the two lines passing through $(1, 0)$ and tangent to $y = x^2 + 3$. Find $\tan(\theta)$.
D) Let A and B be points on $y = x^2 + 3$ such that the tangent lines to $y = x^2 + 3$ at A and B intersect at $(2020, 0)$. Line AB intersects the $y -$axis at $(0, c)$. Find c.

#14 Calculus Team
January Statewide 2022

A) Compute $f^{(13)}(13)$ if $f(x) = x^{12} + \cos(\pi x)$

B) Let $g(x) = 13^x$ and let c be the unique real number in $(1,2)$ such that $g'(c) = \frac{g(2)-g(1)}{2-1}$. Find e^c.

C) Find the maximum possible value of $\sin^{13}(x) + \cos^{13}(x)$.

D) Let $h(x) = 13^{13^x}$. If $h'(1) = 13^a \ln^2(b)$ for positive integers a and b, find $a + b$.

#14 Calculus Team
January Statewide 2022

A) Compute $f^{(13)}(13)$ if $f(x) = x^{12} + \cos(\pi x)$

B) Let $g(x) = 13^x$ and let c be the unique real number in $(1,2)$ such that $g'(c) = \frac{g(2)-g(1)}{2-1}$. Find e^c.

C) Find the maximum possible value of $\sin^{13}(x) + \cos^{13}(x)$.

D) Let $h(x) = 13^{13^x}$. If $h'(1) = 13^a \ln^2(b)$ for positive integers a and b, find $a + b$.