HELPFUL GUIDE TO NOTATION THAT WILL APPEAR ON THIS TEST:

- $[P]$ denotes the area bound by figure P.
- Quadrilateral $ABCD$ is assumed to be the quadrilateral with vertices in that order labelled clockwise unless the question states otherwise
- "NOTA" stands for None of the Above and should be selected if you believe the question is flawed or that the correct answer is not one of the ones presented.

Best of Luck on this test! Good luck and Have Fun!

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1) Unlike most sheep, Sammmmmmy the sheep is a very adventurous sheep and likes to roam. Unfortunately, Sammmmmmy is also a very dumb sheep and if not kept safe he is prone to danger. Hence, Bailey decides to tie Sammmmmmy to his barn. His barn is a perfect rectangular prism with base lengths of 3 and 5 and height of 2. Bailey ties Sammmmmmy's leash to the base of the barn at ground level at a vertex with a rope of length 4. Ignore the dimension of the sheep and the thickness of the rope. Given that the number of square feet of the area on the ground that Sammmmmmy can roam outside the barn can be written in the form $\frac{m\pi}{n}$, compute $m + n.$

 (A) 13   (B) 23   (C) 51   (D) 53   (E) NOTA

2) In Quadrilateral ABCD, $m\angle A = 89°, m\angle B = 94°, m\angle C = 107°$. Given that all measurements are given in degrees, compute $m\angle D$ (in degrees).

 (A) 60   (B) 70   (C) 80   (D) 90   (E) NOTA

3) The summer math camp has just begun! Unfortunately, the Florida weather has proven to be sweltering hot. Eddie, looking to make a few bucks decides to sell some ice cream. The container that Eddie sells his ice cream in is identical to the solid that occurs when a rectangle with sides of lengths 5 and 3 is revolved about one of its longer sides. Compute the volume of Eddie's ice cream container in cubic units.

 (A) 15π   (B) 25π   (C) 45π   (D) 75π   (E) NOTA

4) For some reason or another Sean has become fond of roses. He decides to create a rectangular rose garden. He has 100 feet of fencing with which he can fence the garden. He has decided that one side of the garden will be the wall of his house (to protect the roses) but the other 3 sides will require fencing. It is also given that Sean's house wall is 1000 feet long. Compute the maximum area that Sean's rose garden can be (in square feet)

(A) 625   **(B) 1250**   **(C) 2500**   **(D) 10000**   **(E) NOTA**

5) Consider a parallelogram $ABCD$ with $AB \parallel CD$. Given that the perimeter of $ABCD$ is $18,$ compute the maximum possible value of the sum of the lengths of sides AB and BC.

(A) 9   **(B) 10**   **(C) 11**   **(D) 12**   **(E) NOTA**

6) Some people have favorite food; others have favorite pets. Jake really likes math and hence has a favorite convex quadrilateral! He tells Jeremy about this. Furthermore, Jake tells Jeremy that his quadrilateral has two diagonals one of which has length 10 feet and the other which has length 20 feet. Jake also tells Jeremy that the diagonals of his favorite quadrilateral are perpendicular. Jeremy tells Jake, "I don't know the side lengths of your favorite quadrilateral, but I do know the area which is K feet squared." Assuming Jeremy spoke correctly, compute K.

(A) 50   **(B) 100**   **(C) 200**   **(D) 400**   **(E) NOTA**

7) Refer to Question 6. How many of the following could Jake's favorite quadrilateral <u>possibly</u> be?

- Square
- Rectangle
- Trapezoid
- Kite
- Parallelogram
- Quadrilateral that is not any of the above 5

(A) 6   **(B) 5**   **(C) 4**   **(D) 3**   **(E) NOTA**

8) Consider the quadrilateral bound by the following lines:

$$y = 2x - 3, y = 2x + 12, y = 18 - x, \text{ and } y = 16 - 3x.$$

Which of the following best describes this quadrilateral?

(A)	(B)	(C)	(D)	(E)
PARALLELOGRAM	**CONCAVE**	**KITE**	**TRAPEZOID**	**NOTA**

9) Compute the sum of all possible values of x such that there exists a parallelogram with side lengths 8x+4, 7x+5, 13, and 13.

(A) 1   (B) 2   (C) 3   (D) 4   (E) NOTA

10) A square has semi perimeter of 30 units. What is the area of the square (in units squared)?

(A) 225/4   (B) 225/2   (C) 225   (D) 900   (E) NOTA

11) A rhombus has area 6, and the diagonals have lengths x and y. If its perimeter is $4\sqrt{10}$, give the value of $x + y$.

(A) 4   (B) 6   (C) 8   (D) 10   (E) NOTA

12) Consider cyclic quadrilateral ABCD. Define E to be the intersection point of diagonals $\overline{AC}$ and $\overline{BD}$. If AE=12, EC=7 and BE=6, then compute ED.

(A) 7/2   (B) 6   (C) 7   (D) 14   (E) NOTA

13) Refer to the quadrilateral from question 12. Given that

$\frac{[ACD]}{[ABC]} = \sqrt{\frac{p}{q}}$ where p and q are relatively prime positive integers

then compute $p + 2q$.

(A) 67   (B) 77   (C) 97   (D) 107   (E) NOTA

14) Define $f(k)$ to be the area of the region bound by the graph:

$$|x| + |y| = k.$$

Compute the remainder when $\sum_{k=1}^{2020} f(k)$ is divided by 1000.

(A) 185   (B) 370   (C) 480   (D) 740   (E) NOTA

15) Consider rectangle ABCD with side lengths AB=4 and BC=3. Denote C1 to be the incircle of triangle ADC and C2 to be the incircle of triangle ABC. Let O1 and O2 be the centers of C1 and C2, respectively. Define d to be the distance between O1 and O2. Compute d^2.

(A) 4   (B) 5   (C) 6   (D) 7   (E) NOTA

16) A square has area 200 and is inscribed in a circle. Give the diameter of the circle.

(A) 10   (B) $10\sqrt{2}$ (C) 20   (D) $20\sqrt{2}$ (E) NOTA

17) Vishaka has three points A=(0,3),B=(2,6),C=(5,6), and she knows that she had a fourth one D such that A,B,C,D were the vertices of a parallelogram, but she forgot its coordinates. Find the sum of the x and y coordinates of all possible locations for D.

(A) 15   (B) 16   (C) 17   (D) 18   (E) NOTA

18) Consider a convex quadrilateral ABCD with side lengths AB=12, BC=20, CD= 21, AD = 19. Compute the sum of one exterior angle at each vertex of ABCD (in degrees).

(A) $180°$ (B) $360°$ (C) $720°$ (D) MORE INFO NEEDED   (E) NOTA

19) A rectangle with dimensions 6 by 8 has one diagonal drawn
and two circles inscribed as shown. Find the total area of the
shaded regions (to the left of each circle, inside the
rectangle).
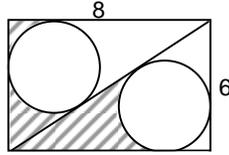
(A) $24 - 2\pi$ (B) $48 - 2\pi$ (C) $24 - 4\pi$ (D) $48 - 4\pi$ (E) NOTA

20) An isosceles trapezoid has legs of length 5 and a base of
length 10. Given that all the dimensions of the trapezoid are
integers, compute the number of distinct possible areas of the
trapezoid.

(A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

21) Four 2 by 2 squares are cut (one at each corner) from a 6 by
10 rectangular piece of cardboard with cuts made perpendicular
to the sides of the rectangle. The four resultant flaps are
folded upward to create an open box. Compute the area of the cut
cardboard used to make this box (in units squared)

(A) 32   (B) 44   (C) 56   (D) 68   (E) NOTA

22) Let $ABCD$ be a kite with side lengths $AB = AD = 5$ and $CD = BC = 13$. If AC has an integral distance, then how many distinct values
exist for $[ABCD]$? (Note: >3 means more than 3 distinct values)

(A) 1   (B) 2   (C) 3   (D) >3   (E) NOTA

23) Consider a rectangle whose perimeter is 18 and area is 8.
Denote l_1 and l_2 to be the lengths of the two diagonals of the
rectangle. Compute $l_1^2 + l_2^2$.

(A) 65   (B) 130   (C) 260   (D) 520   (E) NOTA

24) Consider a cyclic quadrilateral with side lengths 28, 60,
96, and 80. Compute the area of this cyclic quadrilateral.

(A) 4680   (B) 3840   (C) 3552   (D) 2260   (E) NOTA

25) Consider parallelogram $ABCD$. Extend $\overrightarrow{CD}$ past point C to point E such that $CE = 2$. Given that $m\angle ADE = (4x - 3)^\circ$ and $m\angle BCE = (3x + 17)^\circ$, compute the integer closest to x.

(A) 20   (B) 21   (C) 22   (D) 23   (E) NOTA

26) Let S be a square of side length 20202020. A point $P \ni S$ is chosen randomly. Given that the probability that P is closer to the center of the square then one of the vertices can be written in the form m/n where m and n are relatively prime positive integers, then compute m+n.

(A) 3   (B) 5   (C) 10   (D) 11   (E) NOTA

27) A quadrilateral has sides of length $2x + 3, 5x - 2, 4x + 1, x + 1$. Given that the perimeter of the quadrilateral is 39, compute the length of the longest side of the quadrilateral.

(A) 3   (B) 4   (C) 9   (D) 13   (E) NOTA

28) How many of the following statements are true?

- Some trapezoids are parallelograms
- All square are kites
- All parallelograms are concave quadrilaterals
- All rhombi are regular quadrilaterals

(A) 1   (B) 2   (C) 3   (D) 4   (E) NOTA

29) Given that a trapezoid can be partitioned into 3 non-overlapping equilateral triangles of side length 3, compute the area of the trapezoid (in units squared).

(A) $\dfrac{9\sqrt{3}}{4}$ (B) $\dfrac{27\sqrt{3}}{4}$ (C) $\dfrac{81\sqrt{3}}{4}$ (D) $\dfrac{243\sqrt{3}}{4}$ (E) NOTA

30) Which of the following quadrilaterals below MUST have 4 congruent sides?

(A) OCTAGON   (B) RHOMBUS   (C) KITE   (D) CIRCLE   (E) NOTA