1. C $8(4^{2020}) = 2^3 \cdot (2^2)^{2020} = 2^3 \cdot 2^{4040} = 2^{4043}$

2. D $3^x = 29$ so $3 < x < 4$. $5^y = 24$ so $1 < y < 2$. $4^z = 23$ so $2 < z < 3$. From this we see that $y < z < x$.

3. C Since $5^{-x} = 4$, then $5^{3x+2} = (5^{-x})^{-3} \cdot 5^2 = 4^{-3} \cdot 5^2 = \frac{25}{64}$.

4. D The given expression is equal to $(8n^9)^{\frac{1}{3}} = 2n^3$ so $2n^3 = \frac{250}{27}$ and $n = \frac{5}{3}$.

5. B $2^{4x+1} + 8 = 17 \cdot 4^x$ is equivalent to $2(4^x)^2 - 17(4^x) + 8 = 0$. Letting $u = 4^x$: $(2u - 1)(u - 8) = 0$ so $u = 4^x = \frac{1}{2}$ OR $u = 4^x = 8$ for which $x = -\frac{1}{2}, \frac{3}{2}$.
The product of solutions is $-\frac{3}{4}$.

6. B Subtracting the two equations, $1 - 2 = \log(x^2\, y) - \log(xy^2) = \log\left(\frac{x^2 y}{xy^2}\right) = \log\left(\frac{x}{y}\right)$.
Thus, $-1 = \log\left(\frac{x}{y}\right) = \log(x) - \log(y)$.

7. C By the Binomial Expansion Theorem, the constant term in $\left(2y - \frac{1}{y}\right)^4$ is
$\binom{4}{2}(2y)^2\left(-\frac{1}{y}\right)^2 = \frac{4!}{2!2!}(4y^2)(y^{-2}) = 6 \cdot 4 = 24$

8. B The slope of the perpendicular line is the opposite reciprocal slope:
$-\frac{1}{\log_4 9} = -\frac{1}{\frac{\ln(9)}{\ln(4)}} = -\frac{\ln(4)}{\ln(9)} = -\frac{\ln(2^2)}{\ln(3^2)} = -\frac{2\ln(2)}{2\ln(3)} = -\log_3 2 = \log_3 \frac{1}{2}$
using the Change of Base Formula.

9. D $\log x - \log 4 = 1$ implies $\log\left(\frac{x}{4}\right) = 1$ so $\frac{x}{4} = 10$ and $x = 40$.

10. A We need $4 - \frac{|x|}{2} > 0$ so $4 > \frac{|x|}{2}$ and $|x| < 8$. This is the interval (-8, 8).

11. D $I = \frac{E}{R} e^{-\frac{t}{RC}}$ so $\frac{RI}{E} = e^{-\frac{t}{RC}}$. Thus, $\ln\left(\frac{RI}{E}\right) = -\frac{t}{RC}$ and hence $t = -RC\ln\left(\frac{RI}{E}\right)$.

12. E We can repeatedly use the Change of Base: $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \ldots \cdot \log_{31} 32$.
$= \frac{\ln(3)}{\ln(2)} \cdot \frac{\ln(4)}{\ln(3)} \cdot \frac{\ln(5)}{\ln(4)} \cdot \ldots \cdot \frac{\ln(32)}{\ln(31)} = \frac{\ln(32)}{\ln(2)} = \log_2 32 = 5$.

13. D $\log_2 2 \cdot \log_2 4 \cdot \log_2 8 \cdot \log_2 16 = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

14. B Start by finding the prime factorization of $2160 = 2^4 3^3 5^1$. Since this factorization is unique, $x = 4$, $y = 3$, $z = 1$. Thus $2x + 3y + 5z = 2(4) + 3(3) + 5(1) = 22$.

15. C Work from the inside out: $\sqrt[3]{x\cdot\sqrt{x}} = \left(x(x)^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{1}{3}} = x^{\frac{1}{2}}$. So $y = \frac{1}{2}$.

16. B $4^{3x-11} = 4^{-1}$ so by comparing powers, $3x - 11 = -1$. Thus, $3x = 10$ and $x = \frac{10}{3}$.

17. C Let $m = 2^x$ and $n = 2^y$. Then if $\log_m n = k$ for some integer, $m^k = n$ and so $(2^x)^k = 2^{xk} = 2^y$ showing x must divide y. Make a list of each pair (x, y) where $x \le y$ and x divides y: (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10), (2,2), (2,4), (2,6), (2,8), (2,10), (3,3), (3,6), (3,9), (4,4), (4,8), (5,5), (5,10), (6,6), (7,7), (8,8), (9,9), (10,10). This is 10 + 5 + 3 + 2 + 2 + 5 = 27 pairs out of (10)(10) = 100 pairs with replacement. The probability is then 27/100.

18. B $\log_3(\frac{20}{27}) = \log_3(2^2\cdot 5) - \log_3(3^3) = 2\log_3(2) + \log_3(5) - 3\log_3(3)$
or in terms of the variables given, $2a + b - 3$.

19. A Suppose $f(t) = b^t$ for some base b. The given property requires $b^{t+1} = 9b^{t-1}$ or $b^2 = 9$ so b = 3 or -3 (but negative bases are not considered). Since choice (A) can be written $3^{t+2} = 9\cdot 3^t$, this will work. Note the vertical scale change of 9 doesn't change the fact that this property still holds.

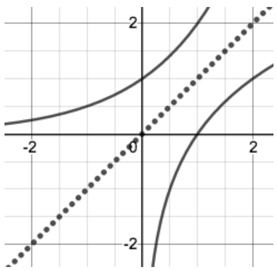
20. C Given: $\log_2 x + \log_4 x + \log_{16} x + \log_{256} x = 15$. Convert to base 2:
$\log_2 x + \frac{\log_2 x}{\log_2 4} + \frac{\log_2 x}{\log_2 16} + \frac{\log_2 x}{\log_2 256} = 15$ or $\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)\log_2 x = 15$.
Thus, $\frac{15}{8}(\log_2 x) = 15$ and so $\log_2 x = 8$ and $x = 2^8 = 256$.
The sum of the digits is 2 + 5 + 6 = 13.

21. C $N = 2^{1234} + 3^{1234} + 4^{1234} + 5^{1234}$.
The units digit of powers of 2 cycle with period 4: 2, **4**, 8, 6, …
The units digit of powers of 3 cycle with period 4: 3, **9**, 7, 1, ….
The units digit of powers of 4 cycle with period 2: 4, **6**, 4, 6, …
The units digit of powers of 5 cycle with period 1: 5, **5**, 5, 5, …
Since 1234 leaves a remainder of 2 when we divide by 4,
we consider the sum 4 + 9 + 6 + 5 = 24 which in turn has units digit 4.

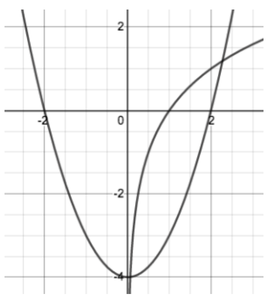
22. C $500 = 1000(1 - e^{-0.15d})$ so $e^{-0.15d} = \frac{1}{2}$ and $-0.15d = \ln\left(\frac{1}{2}\right)$.
This means $0.15\,d = \ln(2) \approx 0.7$ so $d \approx \frac{0.7}{0.15}$ which is between 4 and 5.
The least number of days required is 5.

23. D Let $a = \log_9 x$ and $b = \log_y 8$. Then $a + b = 2$ and $\frac{1}{a} + \frac{1}{b} = 8/3$. Multiplying the two equations, $2 + \frac{a}{b} + \frac{b}{a} = \frac{16}{3}$. Letting $t = \frac{a}{b}$, $3t^2 - 10t + 3 = 0$ so $(3t - 1)(t - 3) = 0$ and $t = \frac{a}{b} = \frac{1}{3}, 3$. With $a = \frac{b}{3}$ we have $a = \frac{3}{2}, b = \frac{1}{2}$ so $x = 9^{\frac{3}{2}} = 27, y = 8^2 = 64$. With $a = 3b$ we have $a = \frac{1}{2}, b = \frac{3}{2}$ so $x = 9^{\frac{1}{2}} = 3, y = 8^{\frac{2}{3}} = 4$. The greatest value of $x + y$ = 27 + 64 = 91.

24. A Note $y = \log_2 x$ and $y = 2^x$ are inverse functions. Since neither of these functions ever intersect with $y = x$, the functions themselves never intersect. This is confirmed via a quick sketch:



25. C Sketching a quick graph, we see there are two intersection points. Based on the concavity of the two increasing functions, there are no additional intersection points.



26. B $\log(\log(\text{googol})) = \log(\log(10^{100})) = \log(100) = 2$

27. D Consider a positive integer k. If $k = 2^q$ for integer q, $\lfloor \log_2 k \rfloor = q$. Otherwise, $k = 2^q + d$ for some $0 < d < 2^q$ and $\lfloor \log_2 k \rfloor = q$ as well. Let's make a table for the first few values of k and $\lfloor \log_2 k \rfloor$:

k	1	2	3	4	5	6	7
$\lfloor \log_2 k \rfloor$	0	1	1	2	2	2	2

We see our sum is of the form of $1(0) + 2(1) + 4(2) + 8(3) + 16(4) + \cdots 2^m(m)$.
Since $\sum_{m=0}^{7} 2^m(m) = 1538$, we need to add 2020 – 1538 = 482 more.
Since $\frac{482}{8} = 60.25$, we need to add 60 more values beyond $2^8 = 256$ so the least n is 60 + 256 = 316.

28. B $g(x) = 2^{1-4x-2x^2} = 2^{3-2(x+1)^2}$ upon completing the square.
The maximum value of the exponent is 3. Since the function $y = 2^x$ is strictly increasing, the maximum of g is $2^3 = 8$.

29. B The first four terms sum to $1 - 2i - 3 + 4i = -2 + 2i$. Due to a shift, every consecutive 4-tuple thereafter has the same sum of $-2 + 2i$. The sum is $2(-2 + 2i) = -4 + 4i$.

30. D For $(x^2 - 9x + 19)^{x^2 - 15x + 56} = 1$, there are three cases to consider:
a) The base $x^2 - 9x + 19 = 1$ so $x^2 - 9x + 18 = (x - 6)(x - 3) = 0$ so x = 3, 6.

b) The exponent $x^2 - 15x + 56 = (x - 7)(x - 8) = 0$ so x = 7, 8. Note these two values of *x* are not also zeros of the base, otherwise, we would have 0^0 .

c) The base $x^2 - 9x + 19 = -1$ and the exponent $x^2 - 15x + 56$ is even. Note the exponent factors as $(x - 7)(x - 8)$ which is the product of two consecutive integers of opposite parity so the exponent is always even.

Then $x^2 - 9x + 20 = (x - 4)(x - 5) = 0$ so x = 4, 5.

In total we see that there are six total solutions with a sum of 3 + 4 + 5 + 6 + 7 + 8 = 33.