**1. C**

**2. D**  so . so . so . From this
 we see that .

**3. C** Since , then .

**4. D** The given expression is equal to so and .

**5. B** is equivalent to . Letting
 so OR for which .
 The product of solutions is .

**6. B** Subtracting the two equations, .

 Thus, .

**7. C** By the Binomial Expansion Theorem, the constant term in is

**8. B** The slope of the perpendicular line is the opposite reciprocal slope:

 using the Change of Base Formula.

**9. D**  implies so and *x* = 40.

**10.** **A**  We need so and . This is the interval (-8, 8).

**11. D** so . Thus, and hence .

**12. E** We can repeatedly use the Change of Base: .
 .

**13. D**

**14. B** Start by finding the prime factorization of . Since this factorization is unique, *x* = 4, *y* = 3, *z* = 1. Thus 2*x* + 3*y* + 5*z* = 2(4) + 3(3) + 5(1) = 22.

**15. C** Work from the inside out:. So .

**16. B**  so by comparing powers, . Thus, and *x* =

**17. C** Let and . Then if for some integer, and so
 showing *x* must divide *y*. Make a list of each pair (*x, y*) where
and *x* divides *y*:(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10), (2,2), (2,4),
 (2,6), (2,8), (2,10), (3,3), (3,6), (3,9), (4,4), (4,8), (5,5), (5,10), (6,6), (7,7), (8,8), (9,9),
 (10,10). This is 10 + 5 + 3 + 2 + 2 + 5 = 27 pairs out of (10)(10) = 100 pairs with
 replacement. The probability is then 27/100.

**18. B**
or in terms of the variables given, .

**19. A** Suppose for some base *b*. The given property requires or
 so *b* = 3 or -3 (but negative bases are not considered). Since choice (A) can be written , this will work. Note the vertical scale change of 9 doesn't change
the fact that this property still holds.

**20. C** Given: Convert to base 2:
 or .
 Thus, and so and .
 The sum of the digits is 2 + 5 + 6 = 13.

**21. C** .
 The units digit of powers of 2 cycle with period 4: 2, **4**, 8, 6, …
 The units digit of powers of 3 cycle with period 4: 3, **9**, 7, 1, ….

 The units digit of powers of 4 cycle with period 2: 4, **6**, 4, 6, …

 The units digit of powers of 5 cycle with period 1: 5, **5**, 5, 5, …
 Since 1234 leaves a remainder of 2 when we divide by 4,
 we consider the sum 4 + 9 + 6 + 5 = 24 which in turn has units digit 4.

**22. C**  so and .
 This means so which is between 4 and 5.
 The least number of days required is 5.

**23. D**  Let *a* = and . Then and . Multiplying
 the two equations, . Letting , so
 and . With we have
 so . With we have
 so . The greatest value of *x* + *y* = 27 + 64 = 91.

**24. A** Note and are inverse functions. Since neither of these functions
 ever intersect with , the functions themselves never intersect. This is confirmed via
 a quick sketch:

 

**25. C** Sketching a quick graph, we see there are two intersection points. Based on the concavity of the two increasing functions, there are no additional intersection points.

 
**26. B**

**27. D** Consider a positive integer *k*. If for integer *q*, . Otherwise,

 for some and as well. Let's make a table
 for the first few values of *k* and

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *k* | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  | 0 | 1 | 1 | 2 | 2 | 2 | 2 |

 We see our sum is of the form of .
 Since , we need to add 2020 – 1538 = 482 more.
 Since , we need to add 60 more values beyond so the least *n* is
 60 + 256 = 316.

**28. B** upon completing the square.
The maximum value of the exponent is 3. Since the function is strictly
increasing, the maximum of *g* is .

**29. B** The first four terms sum to Due to a shift, every consecutive 4-tuple thereafter has the same sum of . The sum is .

**30. D** For , there are three cases to consider:

a) The base so so *x* = 3, 6.

 b) The exponent so *x* = 7, 8. Note these two values of *x* are not also zeros of the base, otherwise, we would have .

 c) The base and the exponent is even. Note the exponent factors as which is the product of two consecutive integers of opposite parity so the exponent is always even.

 Then so *x* = 4, 5.

 In total we see that there are six total solutions with a sum of 3 + 4 + 5 + 6 + 7 + 8 = 33.