Answer Key: BEADB DBDCC BAEDA CBAAB CCDBD BADBC

1. Quadratic formula gives $r, s = -\frac{1}{2}, 2$, so $\lfloor r \rfloor + \lfloor s \rfloor = -1 + 2 = 1$ **(B)**

2. We express 0.231231… as $\frac{231}{999}$, which reduces to 77/333, which is relatively prime, thus we get the desire sum as 410. **(E)**

3. Factoring the polynomial, we get $\ln\big((x-4)(x+4)(x-1)\big)$, which the finite endpoints of the domain's intervals will sum to 1 **(A).**

4. If you cube both sides of $x + y = 6$, you get $x^3 + 3x^2y + 3xy^2 + y^3 = 216$. Subtracting $x^3 + y^3 = 23$, you get $3x^2y + 3xy^2 = 193$. Factoring, you get $3xy(x + y) = 193$, which we can substitute $x + y = 6$, which gives us 193/18 **(D)**

5. $2x - y = 5 \rightarrow y = 2x - 5 \rightarrow x + 2(2x - 5) + 5 = 0 \rightarrow 5x - 5 = 0 \rightarrow x = 1 \rightarrow y = -3$ **(B)**

6. Given $h \in \{3,4,5,6,7,8,11,10,11,12,13\}$, there are 11 possible values.
   - When h = 3 and 4, we have 4 intersection points.
   - When h = 5, we have 3 intersection points.
   - When h = 6, 7, 8, we have 2 intersection points.
   - When h = 11, we have 1 intersection point.
   - When h = 10, 11, 12, 13 we have 0 intersection points.

   Thus the answer is 3 **(D).**

7. This question can be split into three cases: $1^a = 1$, $(-1)^{2k} = 1$, $a^0 = 1$. The real values that are not extraneous are -1, -3, and 2, giving us 6 **(B)**

8. The denominator has a value that is the solution to $x = \sqrt{6 - x} \rightarrow x^2 + x - 6 = 0 \rightarrow (x - 2)(x + 3) = 0 \rightarrow x = 2$, since $x = -3$ is extraneous. The numerator yields a value of $\frac{1+\sqrt{13}}{2}$, given from the equation $y = \sqrt{3 + y}$ using a similar process. The final answer is $\frac{1+\sqrt{13}}{4}$ **(D)**

9. $xy = -3 \rightarrow x = \frac{3}{y} \rightarrow \frac{1}{x} + \frac{1}{y} = \frac{2}{3} = \frac{y}{3} + \frac{1}{y} \rightarrow y^2 - \left(\frac{2}{3}\right)(-3)y - 3 = 0 \rightarrow y = 1, -3$. The equations therefore intersect at (-3, 1) and (1, -3) , so -3+1 = -2 **(C)**

10. Finding the two discriminants of the quadratics, we have $9 - 4k$ and $25 - 8k$. We are looking for values that make $9 - 4k < 0$ and $25 - 8k \geq 0$. Therefore $\frac{9}{4} < k \leq \frac{25}{8}$, which the only integer in that interval is 3 **(C).**

11. The solutions are the integers in the interval $-\frac{8}{3} < x < 6$, which are $x = -2, -1, 0, 1, 2, 3, 4, 5$. So there are 8 **(B)**

12. Given Vieta's formulas, the polynomial's roots must have a product of 1, and a pairwise sum of the solutions as A, and a sum of solutions of -7/3. Using these facts, you can discern that the sequence is -3, 1, -1/3, which would cause the linear coefficent to be -7 **(A)**

13. Two non-negative values can only add to zero if they are both zero. But, these two quadratics do not have the same roots, so the answer is **(E)**

14. Upon combining the subtraction of logs as division, and exponentiation we have $\frac{x^2+1}{2x-1} = 4$. This has values of $4\pm\sqrt{11}$, which, neither are extraneous so the answer is 8 **(D)**.

15. The partial fraction decomposition of $\frac{5x-7}{(x-1)^3} = \frac{0}{x-1} + \frac{5}{(x-1)^2} - \frac{2}{(x-1)^3}$, so the answer is 3 **(A)**

16. $d = \sqrt{(2021 - 2024)^2 + (1337 - 1333)^2} = \sqrt{(-3)^2 + 4^2} = 5$ **(C)**

17. The two conics can be put into standard form from completing the square: $\frac{(x-2)^2}{16} + \frac{(y-4)^2}{25} = 1$ and $(x-2)^2 + (y-2)^2 = 9$. The area of the ellipse is 20π and the circle is 9π, giving us 11π **(B)**

18. Given arithmetic sequences, $a_5 = 10$, $a_{13} = 19 = 10 + 8d$. Therefore the common difference is 9/8 **(A)**

19. After squaring and rearranging, we find that $\sqrt{21 + 12\sqrt{3}} = \sqrt{12} + \sqrt{9}$, which $A^2 - B^2 = 144 - 81 = 63$ **(A)**

20. The coefficient of x^3 would be calculated by $10(2x^3)^2\left(\frac{1}{x}\right)^3 = 40x^3$ **(B)**

21. The inequality produces a rhombus with vertices of $(\pm 47, 0)$ *and* $(0, \pm 43)$. The area of a rhombus is $A = \frac{1}{2}d_1 d_2$, which in this case equals $\frac{1}{2}\big(2(47)\big)\big(2(43)\big) = 4042$ **(C)**

22. Equations (1), (2), and (5) are even functions. **(C)**

23. Let our complex numbers be $m = x + yi$ and $n = w + zi$. Given their sum and difference, we arrive at the system of equations $m + n = 2 - 8i$ and $m - n = 9 + 3i$. Squaring the sum $(m+n)^2 = m^2 + n^2 + 2mn = -60 - 32i$ and $(m-n)^2 = m^2 + n^2 - 2mn = 72 + 54i$. Adding these equations gives us $2m^2 + 2n^2 = 12 + 22i$, which makes $m^2 + n^2 = 12 + 22i$, giving the desired sum of 34. **(D)**

24. $-\frac{4}{2-x} \geq \frac{1}{3x^2} \rightarrow 0 \geq \frac{12x^2+2-x}{3x^2(2-x)}$. The numerator is always positive so this will be true only when $x > 2$. **(B)**

25. The distinct arrangements of ANAGHS is equal to 6!/2! = 360. The number of permutations of ALANWU with the L directly between the A's would be done by grouping ALA as one letter and permuting, 4! = 24. The desired answer is 360 – 24 = 336 **(D)**

26. Triangle Inequality Theorem requires $4 < x < 26$, therefore the only correct answer is $\frac{18}{\sqrt{2}}$ **(B)**

27. Solving each inequality gives respectively solutions of $x \leq -2 \; or \; x \geq 10$ and $-6 < x < 0$. This means they share the solutions of -3, -2, and -1. **(A)**

28. Let A = Angelina's work rate, B = Luka's work rate, and C = Alan's work rate. We then set up a system of equations based on the values, $A + B = \frac{1}{2}$, $A + C = \frac{5}{12}$ and $B + C = \frac{1}{4}$. Therefore $A + C - A - B = C - B = \frac{5}{12} - \frac{1}{2} = -\frac{1}{12} \rightarrow C + B + C - B = 2C = \frac{1}{6} \rightarrow C = \frac{1}{12} \rightarrow B = \frac{1}{6} \rightarrow A = \frac{1}{3} \rightarrow A + B + C = \frac{7}{12}$. So, we get 12/7 hrs. **(D)**

29. An eighth of the determinant of matrix A multiplies to $\frac{1}{8}x^2 - \frac{5}{8}x - 2$. Any infinite geometric series will have a finite sum given that the common ratio $|r| < 1$. This parabola is between -1 and 1 for $-3 < x < -\frac{51}{40}$ and $\frac{251}{40} < x < 8$. Thus, the answer is -2 (**B)**

30. If one enumerates the recursive series, the first number that occurs that is non-prime is 95. **(C)**