Answers:

0. **2021**

1. **20**

2. **1.677**

3. **4.5247**

4. **2.088**

5. **11.7082**

6. **7.4666**

7. **95.489**

8. **95.748**

9. **– 0.434**

10. **183**

11. **13**

12. **2021**

13. **45.25**

14. **7.1548**

0. Answers: A = 60, B = 40, C = 90, D = 200, Summary = 2021

1. Answers: $A = \frac{1}{4}, B = \frac{1}{10}, C = \frac{2}{5}, D = \frac{1}{5}, \ Summary = 20$

Solutions: $A = P(Teacher\ Y) = \frac{50}{200} = \frac{1}{4}$. $B = P(Teacher\ Z\ \cap 3) = \frac{20}{200} = \frac{1}{10}$. $C = P(5 \cup Teacher\ Y) = \frac{36+50-6}{200} = \frac{80}{200} = \frac{2}{5}$. $D = P(2 \mid Teacher\ X) = \frac{12}{60} = \frac{1}{5}$. $Summary = \frac{19}{\frac{1}{4}+\frac{1}{10}+\frac{2}{5}+\frac{1}{5}} = \frac{19}{\frac{19}{20}} = 20$.

2. Answers: A = 0.693, B = 0.172, C = 0.626, D = 0.186, Summary = 1.677

Solutions: Part A is a binomial setting with $n = 25\ and\ p = \frac{48+36+36}{200} = 0.6$. Letting X represent the number of success in the sample we have $\mu_x = E(X) = 25(0.6) = 15\ and\ \sigma_x = SD(X) = \sqrt{25(0.6)(0.4)} \approx 2.45$. One standard deviation below the mean is 15 – 2.45 = 12.55 and one standard deviation above the mean is 15 + 2.45 = 17.45. Since the binomial is discrete, we are looking for $P(13 \le X \le 17) = binomcdf(25, 0.6, 17) - binomcdf(25, 0.6, 12) \approx 0.693$. Part B is a hypergeometric setting with an expected number of successes of E(X) = 0.6(25) = 15; so $B = P(X = 15) = \frac{\binom{80}{10}\binom{120}{15}}{\binom{200}{25}} \approx 0.172$. Part C is a binomial with $n = 20\ and\ p = \frac{18+8+6}{50} = 0.64$ that is preceded by the random selection of a particular teacher: $P(Teacher\ Y) = \frac{1}{3}$. $C = \frac{1}{3} \times binompdf(20, 0.64, 10) \approx 0.626$. Part D is a hypergeometric probability from only Teacher X: $D = \frac{\binom{27}{10}\binom{33}{10}}{\binom{60}{20}} \approx 0.186$. Summary = 0.693 + 0.172 + 0.626 + 0.186 = 1.677.

3. Answers: A = 0.0106, B = 0.0108, C = 3.0556, D = 1.4477, Summary = 4.5247

Solutions: Part A is a multinomial setting since we are sampling with replacement and Part B is a hypergeometric setting since we are sampling without replacement: $A = \frac{30!}{10! \times 10! \times 10!}(0.30)^{10}(0.25)^{10}(0.45)^{10} \approx 0.0106$ when rounded and B = $\frac{\binom{60}{10}\binom{50}{10}\binom{90}{10}}{\binom{200}{30}} \approx 0.0108$ when rounded. Entering the scores 1, 2, 3, 4, and 5 into L_1 and the score frequencies of Teacher X into L_2, Teacher Y into L_3, Teacher Z into L_4, and the total combined score frequencies into L_5 is helpful for this question and many to follow. Running the *1-Variable Statistics* program on L_1 and each of the frequency lists in L_2, L_3, and L_4 one at a time gives us the following: $\bar{X} = 2.85, S_X \approx 1.4477, \bar{Y} = 2.84, S_Y \approx 1.2675, \bar{Z} \approx 3.0556, and\ S_Z \approx 1.4011$. This makes C = 3.0556 and D = 1.4477 since they are the highest mean and largest standard deviation, respectively. Summary = 0.0106 + 0.0108 + 3.0556 + 1.4477 = 4.5247.

4. Answers: A = 907,200, B = 1,814,400, C = 0.994, D = 0.05, Summary = 2.088

Solutions: Part A describes a STRATIFIED random sample: $A = \frac{10!}{2! \times 2!} = 907{,}200$. Part B describes a MULTISTAGE random sample: $A = \frac{10!}{2!} = 1{,}814{,}400$. Part C is a sampling distribution of the sample mean calculation which allows for using the normal distribution for the calculation of the requested probability since the conditions for using the Central Limit Theorem are met: $C = normalcdf(2.5, 3.5, 2.94, 1.38/\sqrt{64}) \approx 0.994$. NOTE: Rounding the standard deviation used in the calculation (1.38) to at least the hundredths place will result in the same correct final answer when rounded to the thousandths place. $D = \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.60(1-0.60)}{96}} = 0.05$. Summary = $\frac{1{,}814{,}400}{907{,}200}(0.994 + 0.05) = 2.088$.

5. Answers: A = 8.7456, 2.3802, C = 0.1655, D = 0.4169, Summary = 11.7082

Solutions: $A = \mu_{X+Y+Z} = \mu_X + \mu_Y + \mu_Z = 2.85 + 2.84 + 3.0556 = 8.7456$. $B = \sigma_{X+Y+Z} = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2} = \sqrt{1.4477^2 + 1.2675^2 + 1.4011^2} \approx 2.3802$ when rounded. Since random variables X, Y, and Z are all normally distributed, so is the random variable formed by their sum with the mean and standard deviation that were calculated in parts A and B, respectively: $C = P(8 \le X + Y + Z \le 9) = normalcdf(8, 9, 8.7456, 2.38) \approx 0.1655$. NOTE: Rounding the standard deviation used in the calculation (2.38) to at least the hundredths place will result in the same correct final answer when rounded to four decimal places. Since X, Y, and Z are all independent and normally distributed, $D = P(X > 2\ and\ Y > 2\ and\ Z > 2) = P(X > 2) \times P(Y > 2) \times P(Z > 2) = normalcdf(2, 999999, 2.85\ , 1.4477) \times normalcdf(2, 999999, 2.84, 1.2675) \times normalcdf(2, 999999, 3.0556, 1.4011) \approx (0.7214)(0.7462)(0.7744) \approx$ 0.4169 when rounded. NOTE: Rounding each of the normal distribution calculations to at least four decimal places results in the same correct final answer when rounded. Summary = 8.7456 + 2.3802 + 0.1655 + 0.4169 = 11.7082.

6. Answers: A = 0.0384, B = 0.9744, C = 6.25, D = 0.2038, Summary = 7.4666
Solutions: Parts A and B are geometric settings with $p = 0.60$: $A = P(X = 4) = (0.40)^3(0.60) = 0.0384$ and $B = P(X \leq 4) = 1 - (0.40)^4 = geometcdf(0.6, 4) = 0.9744$. Part C is a geometric setting with $p = 0.4$: $\mu_G = \frac{1}{p} = \frac{1}{0.4} = 2.5$ and $\sigma_G^2 = \frac{1-p}{p^2} = \frac{0.6}{(0.4)^2} = 3.75$. Their sum is C = 2.5 + 3.75 = 6.25. Part D is a negative binomial setting with probability of success $p = \frac{18+8+6}{50} = 0.64$ and n = 5 trials which is just a binomial probability calculation with n – 1 trials multiplied by the probability of one final success on trial 5: $D = binompdf(4, 0.64, 2) \times 0.64 \approx 0.2038$.
Summary = 0.0384 + 0.9744 + 6.25 + 0.2038 = 7.4666.

7. Answers: A = 0.126, B = 92, C = 2.447, D = 0.916, Summary = 95.489
Solutions: $A = ME = Z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{0.55(0.45)}{60}} \approx 0.126$ when rounded. Given (0.5211, 0.7588), the margin of error is $ME = \frac{0.7588-0.5211}{2} = 0.11885$ and so $Z^* = \frac{0.11885}{\sqrt{\frac{0.64(0.36)}{50}}} \approx 1.751$ which makes $B = normalcdf(-1.751, 1.751, 0, 1) \approx 0.92 = 92\%$ when rounded to the nearest integer percent. Running the *1-Proportion Z-Test* program with x = 55, n = 90, and the two-tail alternative selected, we get a test statistic of Z = 2.432 and p-value of p = 0.015 when rounded. This makes C = 2.432 + 0.015 = 2.447. In order to reject $H_0: P_T = 48.3\%$ and support $H_A: P_T > 48.3\%$ at a 2.5% level of significance, we need $\hat{p} \geq invNorm\left(0.975, 0.483, \sqrt{\frac{0.483(1-0.483)}{200}}\right) \approx 0.552255$. If we assume that $P_T = 0.60$ is true, then the power of the test is $D = normalcdf\left(0.552255, 999999, \sqrt{\frac{0.60(1-0.60)}{200}}\right) \approx 0.916$.
Summary = 0.126 + 92 + 2.447 + 0.916 = 95.489.

8. Answers: A = 0.748, B = 91, C = 4, D = 0, Summary = 95.748
Solutions: The width of a confidence interval is twice the margin of error. The critical t-score for a 95% confidence with n – 1 = 59 degrees of freedom is $t^* = invT(0.975, 59) \approx 2.001$. This makes the width of the interval $A = 2 \times t^* \frac{s_x}{\sqrt{n}} = 2 \times 2.001 \frac{1.4477}{\sqrt{60}} \approx 0.748$ when rounded. Alternatively, running the *T-Interval* program on Teacher X's summary statistics produces an interval of (2.476, 3.224) and subtracting these limits also gives us the correct answer of A = 0.748. The margin of error of the confidence interval is $ME = \frac{3.15-2.53}{2} = 0.31$ which gives a critical t-score of $t^* = \frac{ME}{s_y/\sqrt{n}} = \frac{0.31}{1.2675/\sqrt{50}} \approx 1.7294$ which makes $B = tcdf(-1.7294, 1.7294, 59) \approx 0.91 = 91\%$ when rounded correctly. Running the *T-Test* program once at a time on each set of summary statistics produces the results summarized in the table below:

Teacher	Test Statistic	Right-Tail P-Value
X	1.7657	0.04131
Y	1.7852	0.04021
Z	3.6265	0.00024
Total	4.3023	0.00001

Note, however, that it is really only necessary to perform the one-sample t-test on Teacher X and Teacher Y since the scores of Teacher Z and the Total combined scores both have an even higher mean and are each based upon a much larger sample size which will undoubtedly lead to even smaller p-values than the first two tests. Thus, C = 4. Running the *T-Test* program once at a time on each set of summary statistics produces the results summarized in the table below:

Teacher	Test Statistic	Two-Tail P-Value
X	– 0.1070	0.9151
Y	– 0.1674	0.8678
Z	1.2567	0.2122
Total	0.7170	0.4742

This time, it is really only necessary to run the t-test on Teacher Z's summary statistics since the other 3 each have a mean even closer to 2.87 which will lead to even larger p-values. Thus, D = 0. Summary = 0.748 + 91 + 4 + 0 = 95.748.

9. Answers: A = – 0.956, B = 0.183, C = 0.339, D = 0, Summary = – 0.434
Solutions: Running the *2-Proportion Z-Test* program on Teacher X vs. Teacher Y gives us a test statistic of Z = –0.965 and a left-tail p-value of p = 0.1695 and a two-tail p-value of p = 0.3391. This makes A = –0.965 and C = 0.339 since each of the other two possible pairwise comparisons of pass rates will lead to larger two-tail p-values because the pass rates being compared are even closer together. Just for the record, the other two two-tail p-values are 0.4565 when comparing Teacher X's pass rate with Teacher Z's and 0.7356 when comparing Teacher Y's pass rate with Teacher Z's. $B = 1.96\sqrt{\frac{0.55(0.45)}{60} + \frac{0.64(0.36)}{50}} \approx 0.183$ and D = 0 because the smallest p-value of the three possible pairwise comparisons is well over 0.05. Summary = –0.956 + 0.183 + 0.339 + 0 = –0.434.

10. Answers: A = 0.039, B = 0.863, C = 0.928, D = 0, Summary = 183
Solutions: Running the *2-Sample T-Test* program on each of the three possible pairs of hypotheses listed in parts A, B, and C gives us the following three test statistics (in absolute value): A = t = 0.039, B = t = 0.863, and C = t = 0.928. Since each of these test statistics is less than 1 and the associated p-values are all well over 0.05, none of the test results are statistically significant at the 0.05 level. So, D = 0. Summary = 100(0.039 + 0.863 + 0.928 + 0) = 183.

11. Answers: A = 6, B = 3, C = 4, D = 0, Summary = 13
Solutions: Although it is not necessary to perform all four $\chi^2\ Tests$, we will do so for the sake of thoroughness. The observed and the expected counts for each teacher and the Total scores are summarized below. Each expected count is determined by multiplying the total observed count in each column by each score's percentage in the global distribution.

	Teacher X		Teacher Y		Teacher Z		Total Scores	
Score	Observed	Expected	Observed	Expected	Observed	Expected	Observed	Expected
1	15	12.84	10	10.7	15	19.26	40	42.8
2	12	11.76	8	9.8	20	17.64	40	39.2
3	10	15.96	18	13.3	20	23.94	48	53.2
4	13	10.8	8	9	15	16.2	36	36
5	10	8.64	6	7.2	20	12.96	36	28.8
Total	60	60	50	50	90	90	200	200

We can now run a chi-square goodness of fit test on each pair of observed and expected counts to obtain the four test statistics either by entering the requisite pair of observed counts and expected counts into a pair of lists and running the $\chi^2\ GOF - Test$ program or by using the formula $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$. Doing so gives us the following four test statistics for Teacher X, Teacher Y, Teacher Z, and the Total scores, respectively: 3.2561, 2.3484, 5.8195, 2.5078 and the p-values associated with each of the four tests are 0.516, 0.672, 0.213, and 0.643, respectively. This makes A = 5.8195 + 0.213 = 6 and B = 2.3484 + 0.672 = 3 when each are rounded to the nearest integer. The degrees of freedom for each of these tests is the number of categories minus 1; so, C = 5 – 1 = 4. Since the smallest of the four p-values (0.213) is not below 0.05, none of the others are below 0.05 as well. This makes D = 0. Summary = 6 + 3 + 4 + 0 = 13.

12. Answers: A = 10, B = 9, C = 8, D = 3, Summary = 2021
Solutions: The expected count of Teacher Y's scores of 2 in this chi-square two-way table test is $A = \frac{50(40)}{200} = 10$. Entering the set of observed scores into a 3 × 5 matrix in the calculator and running the $\chi^2 - Test$ program gives us a test statistic of $\chi^2 \approx 8.93395$ which rounds to B = 9. The degrees of freedom for this test are the number of rows minus 1 times the number of columns minus 1: C = (3 – 1)(5 – 1) = 8. The p-value of the test is p = 0.3479 which is not less than 0.05, so D = 3. Summary = $(10^2 + 1)(9 + 8 + 3) + 1 = 101(20) + 1 = 2021$.

13. Answers: A = 10.75, B = 10, C = 9.5, D = 15, Summary = 45.25
Solutions: Entering the 20 numbers into a list and running the *1-Variable Statistics* program gives us $A = \bar{X} = 10.75$, $B = Q_2 = 10, C = IQR = 15 - 5.5 = 9.5, and\ D = \frac{20+15+10}{3} = 15$. Summary = 10.75 + 10 + 9.5 + 15 = 45.25.

14. Answers: A = 6.1548, B = 0, C = 0, D = 1, Summary = 7.1548
Solutions: The *1-Variable Statistics* program output gives us $A = S_X = 6.1548$, B = 0 (because no data values are outside of the lower or upper fences), C = 0, and D = 1 (because the mean of the set of Z-scores is 0 and the standard deviation is 1). Summary = 6.1548 + 0 + 0 + 1 = 7.1548.